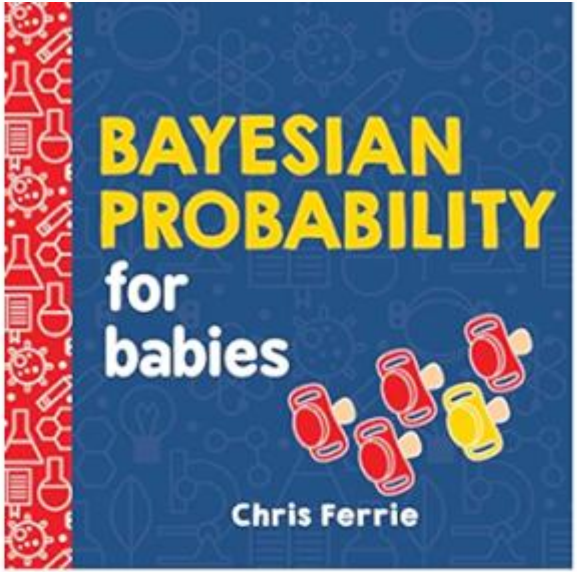
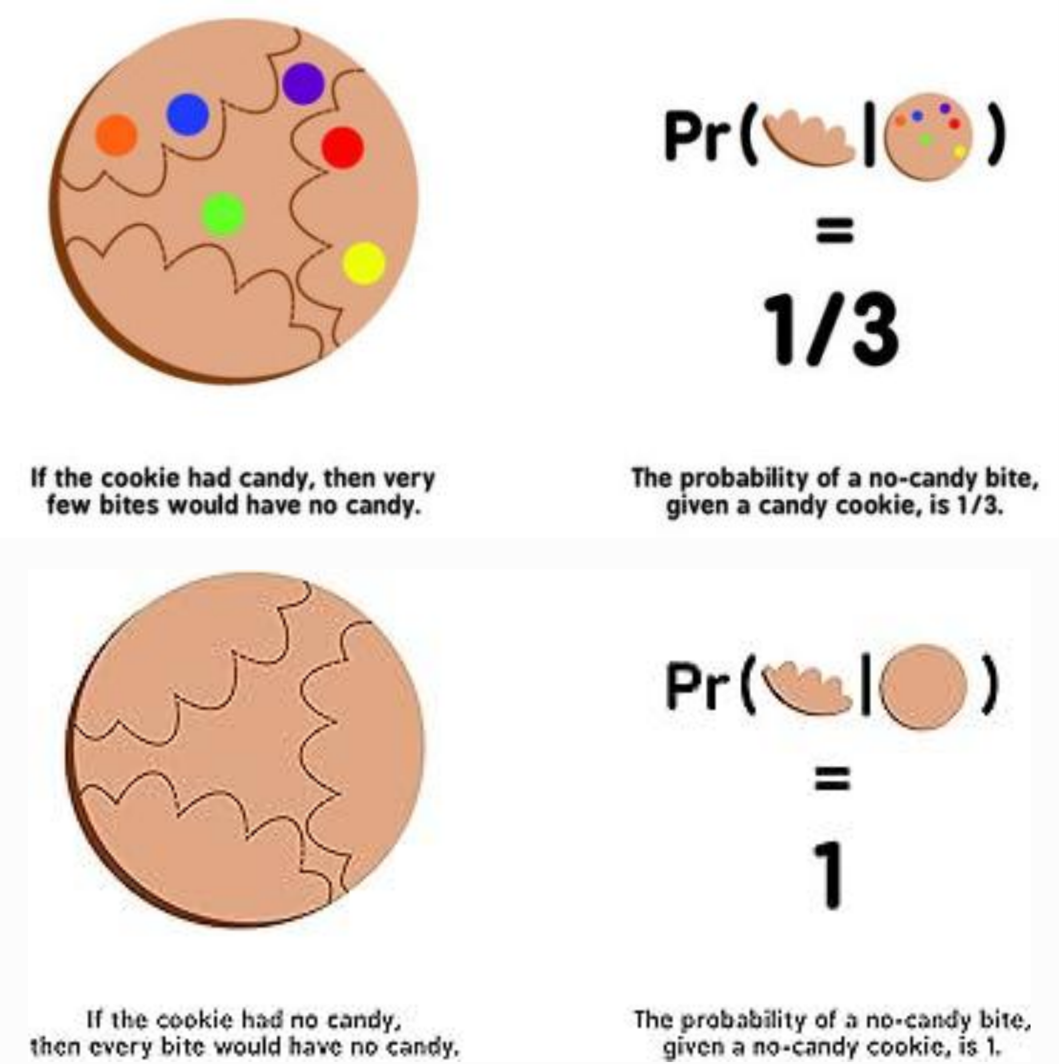

BAYESIAN
PROBABILITY
for
babies
Chris Ferrie


Pr(|)
=
1/3
If the cookie had candy, then very few bites would have no candy.
The probability of a no-candy bite, given a candy cookie, is 1/3.
Pr(|)
=
1
If the cookie had no candy, then every bite would have no candy.
The probability of a no-candy bite, given a no-candy cookie, is 1.

CS 1671 / CS 2071
Human Language Technologies

Session 4: Probability and linear algebra review

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September 3, 2026

University of Pittsburgh | School of Computing and Information

Course logistics

- Homework 1 has been released.
  Is **due Sep 17**
- Please remind me of your name before asking or answering a question

Overview: Linear algebra and probability review

1. Preprocessing coding activity
2. Probability review
3. Linear algebra review

Copying the class Python environment

After installing Pixi, download the **new** pyproject.toml file

- Create a new directory called whatever you like
    - **mkdir class-env**
    - **cd class-env**
- Download the ***new*** manifest file that lists all packages in the class environment
    - I removed the problematic ‘huggingface’ channel
    - pyproject.toml
    - Alternative link: https://github.com/michaelmilleryoder/cs1671_fall2026_jupyterhub/blob/main/pyproject.toml

Install packages

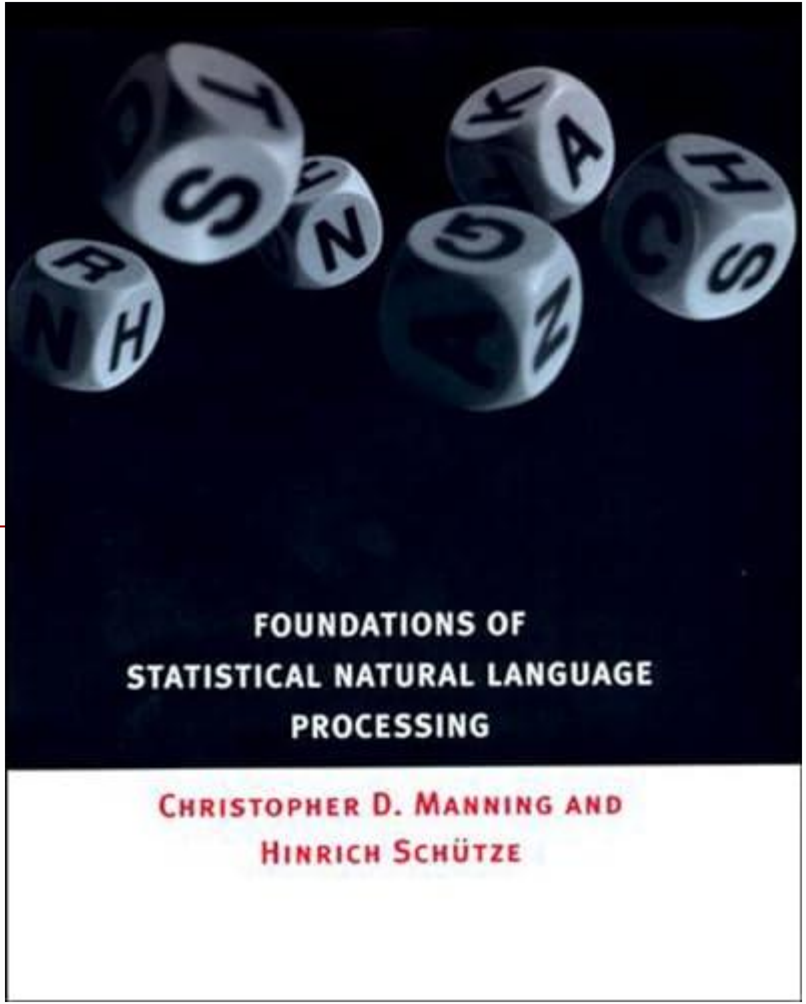
- Install packages in the class environment
  - **pixi install**
- This will take awhile! In the meantime, you can view what's in the pyproject.toml "manifest" file in a text editor
- Test whether you have the class environment installed
  - **pixi run python --version**
  - This should display Python 3.11.16

Coding activity:
Preprocessing Airbnb listings

Coding activity: preprocessing Airbnb listings

1. Go to this nbgitpuller link (also available on course website)
2. Log in with your Pitt username if necessary
3. Start a server with **TEACH – 6 CPUs, 48 GB**
4. Load custom environment at **/ix1/cs1671-2026f/class-env/.pixi/envs/default**
5. This should pull the cs1671_fall2026_jupyterhub folder into your JupyterLab
6. Open **session3_preprocessing.ipynb**

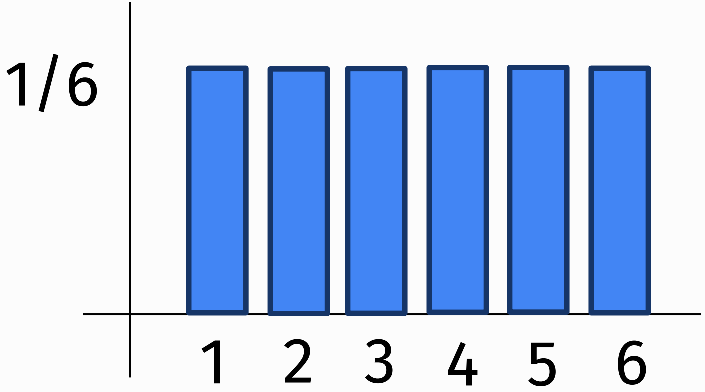
Probability review

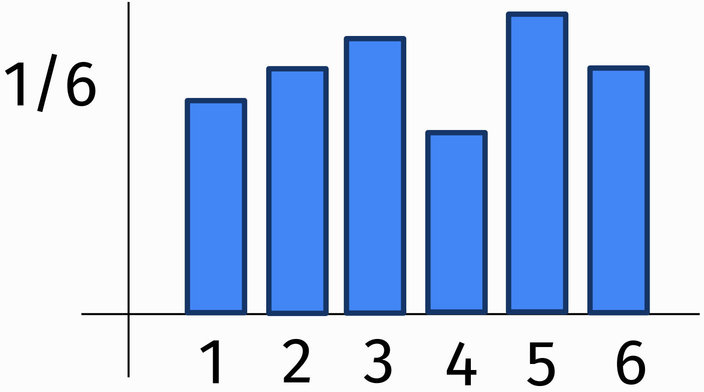


Probability

- Probability of an event a occurring
- $P(a)$
  - For example, a could be a die showing a 2 out of $\{1, 2, 3, 4, 5, 6\}$
- Estimate $P(a)$ as $\dfrac{\text{count}(a)}{\text{count}(\textit{all events})}$
  - Relative frequency or maximum likelihood estimate (MLE)

Probability distributions





Random variables

- **Random variable:** a variable that can hold multiple values according to a probability distribution
  - Probability distribution of values can be estimated empirically
  - For example, flipping a coin multiple times (possible outcomes {H, T}) and recording the result as 0 for tails and 1 for heads
- Distribution of a random variable *X*
  - *P(X)* is a probability distribution over all possible values in the sample space. Probability mass function
  - *P(X = x)* is the probability that the random variable *X* has the value *x*
  - *P(X = heads),* where *X* is the random variable of a coin flip

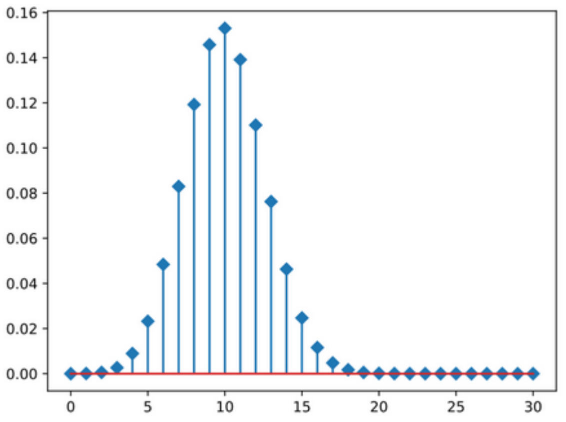


Figure 7.1: $\mathbf{P}(k \text{ heads})$ in 30 tosses, success prob 1/3.

Figure from Baxter 2023

Joint probability

- Probability of 2 events both occurring

$$P(A \cap B)$$

$$P(A, B)$$

- When rolling 2 dice, what's the probability of getting two 5s?

Let D_1 be dice 1, D_2 be dice 2. These events are independent, so:

$P(D_1 = 5, D_2 = 5) = P(D_1 = 5) \cdot P(D_2 = 5)$

$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$ since there are 36 different possible combinations

Conditional probability

- Probability distributions sometimes change if you know another event has occurred or not occurred
- **Conditional** probability of an event *a* occurring **given that another event, *b*, has already occurred**
  - $P(a|b)$

Conditional probability

- Assume
  - X is the outcome of rolling a die once
  - F is the event $X = 6$
  - E is the event $X > 4$
- Die is rolled and we are told that E has occurred
- What is $P(F|E)$, that is, $P(X=6|X>4)$?

Adapted from Roussas 2007

Conditional probability

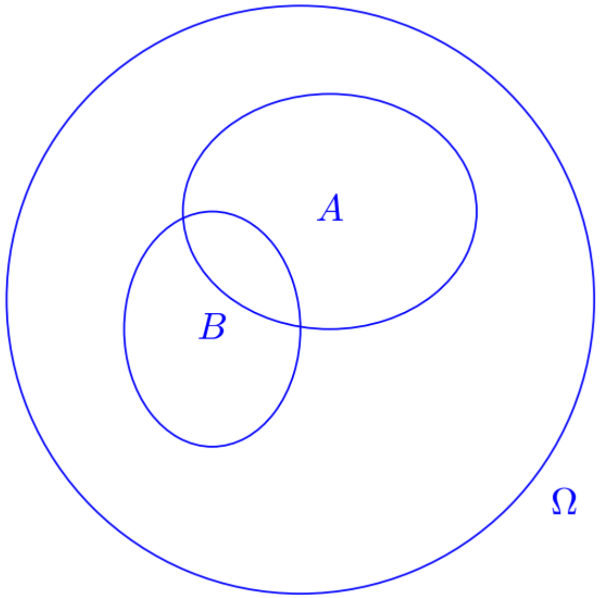


Figure 4.1: Events on the dart board

- Assume a very bad dart thrower (maybe Michael)

$$\mathbf{P}(A) = \frac{\mathbf{area}(A)}{\mathbf{area}(\Omega)}$$

Adapted from Baxter 2023

Conditional probability

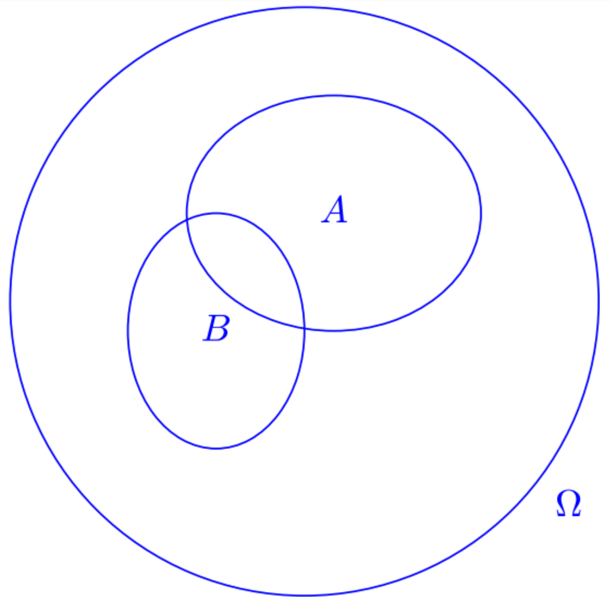


Figure 4.1: Events on the dart board

- You don't see the throw, but somebody tells you that the dart landed in B (so B occurred)
- What is the formula for P(A|B)?

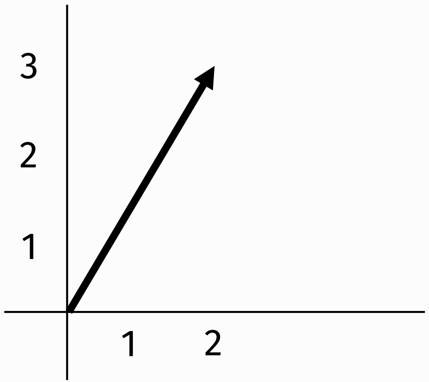
Linear algebra review

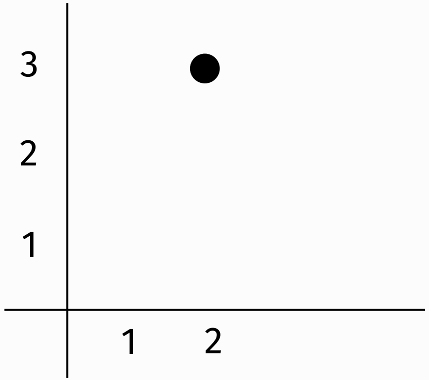
Vectors

An array of numbers with D dimensions

$$[\ 2\ 3\ ]$$

can be represented as a point in D-dimensional space





Dot product: vector · vector

Sum of the products of each vector dimension

$$\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \cdots & v_N \end{bmatrix} \quad \bullet \quad \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

$$\mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \cdots + v_N w_N$$

Matrices

A matrix is an array of numbers

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

Two rows, three columns.

Slide adapted from David Mortensen

It's Easy to Multiple a Matrix by a Scalar

$$2 \cdot \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 \cdot 5 & 2 \cdot 2 \\ 2 \cdot 3 & 2 \cdot 1 \end{bmatrix} = \begin{bmatrix} 10 & 4 \\ 6 & 2 \end{bmatrix}$$

Slide adapted from David Mortensen

Dot product: vector · matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

Dot product: matrix · matrix

Let a_1 and a_2 be the row vectors of matrix A and b_1 and b_2 be the column vectors of a matrix B. Find $C = AB$

$$\begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} a_1 \cdot b_1 & a_1 \cdot b_2 \\ a_2 \cdot b_1 & a_2 \cdot b_2 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

A must have the same number of rows as B has columns.

Slide adapted from David Mortensen