Markov jokes:

Once you’ve heard the latest one, you’ve heard them all.

CS 2731 Introduction to Natural Language Processing

Session 19: HMMs part 2, Viterbi algorithm, neural sequence labeling

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Course logistics: homeworks

- Homework 3 grades released
- Homework 4 is **due Mon Mar 25**
  - Part 1: Do part-of-speech tagging manually with the Viterbi algorithm
  - Part 2: Fine-tune BERT-based models for part-of-speech tagging in English and Norwegian
    - Copy and fill in a skeleton Colab notebook

Course logistics: project

- Project peer review **due Wed Mar 27**
  - Was released today
  - Form where you will review your own and your teammates' contributions so far
  - Will not be used for grading, just for addressing any issues
- Basic working system **due Thu Apr 4**

Overview: HMMs part 2, Viterbi alg, neural sequence labeling

- HMMs review
- Training HMMs
- Decoding HMMs: Viterbi algorithm
- Sequence labeling with RNNs and transformers

HMMs review

HMM review

With a partner, review:

1. What are the 2 key assumptions that HMMs make?
2. What are the 2 key tables of probabilities in HMMs and what do they mean?

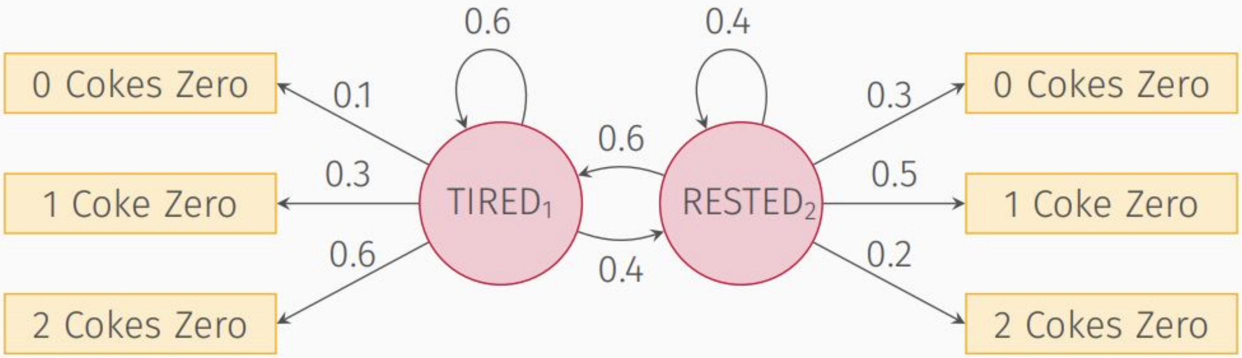
A formal definition of the Hidden Markov Model (HMM)

$Q = q_1, \ldots, q_N$	a set of N states
$A = a_{1,1}, a_{1,2}, \ldots$	a transitional probability matrix of cells a_{ij}, where each cell is a probability of moving from state i to state j. $\sum_{j=1}^{N} a_{ij} = 1 \; \forall i$
$O = o_1, \ldots, o_T$	a sequence of T observations, each drawn from a vocabulary V.
$B = b_1, \ldots, b_n$	a sequence of observation likelihoods (or emission probabilities). The probability that observation o_t is generated by state q_i.
$\pi = \pi_1, \ldots, \pi_N$	an initial probability distribution over states (the probability that the Markov chain will start in state q_i. Some states q_j may have $p_j = 0$ (meaning they cannot be initial states). $\sum_{i=1}^{N} \pi_i = 1 \; \forall i$

Slide credit: David Mortensen

The Coke Zero Example

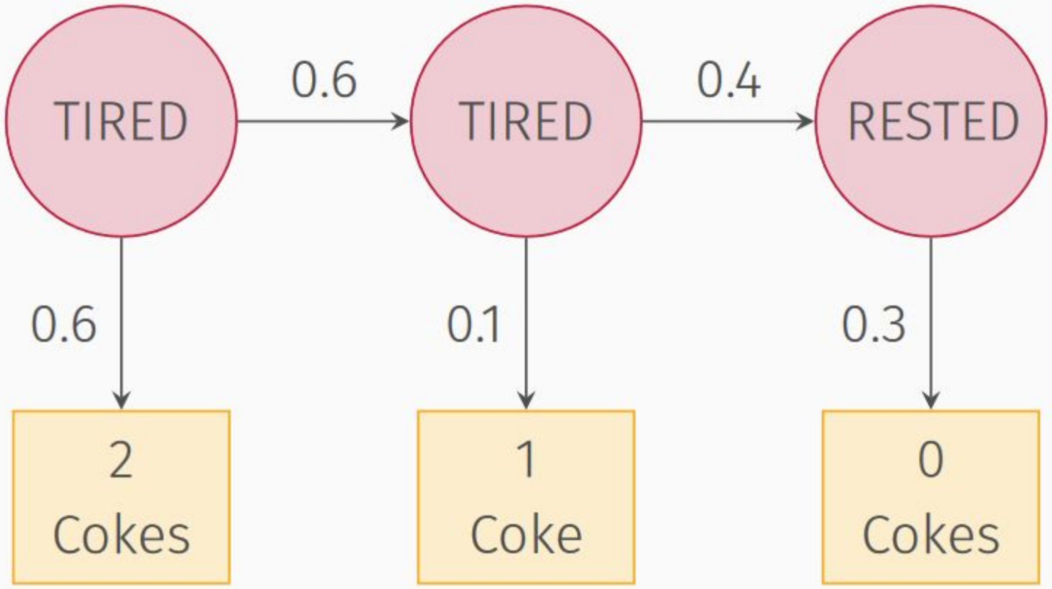
Since I do not drink coffee, I must drink Coke Zero to remain caffeinated. My consumption is related to my exhaustion. Could you build a model to infer my exhaustion from the number of Coke Zero bottles added to my wastebasket each day?



$$\pi = [0.7, 0.3]$$

Slide credit: David Mortensen

An example HMM sequence



Slide adapted from David Mortensen

Training HMMs

Training an HMM

How do we learn the transition and emission probabilities?

- If we have (enough) data labeled with hidden and observed events, can just **use MLE/relative frequencies** with or without smoothing
- If we don't have (enough) labeled data, can use the **Forward-Backward Algorithm**, a special case of the Expectation Maximization (EM) algorithm
  - We won't go into the details of this algorithm, but the overview is that you start with an initial estimate and use that estimate to compute a better one iteratively

Training HMMs with labeled data

Suppose we knew both the sequence of days in which a grad student is tired or rested and the number of Cokes Zero that she consumes each day:

0 rested	3 tired	1 rested
1 tired	2 tired	2 tired
0 rested	0 rested	2 rested

How would you train an HMM?

Slide adapted from David Mortensen

Using MLE to train HMMs

First, compute π from the initial states:

$$\pi_t = 1/3 \; \pi_r = 2/3$$

The we can compute the matrix A:

$$\begin{array}{ll} p(tired|tired) = 1/2 & p(tired|rested) = 1/6 \\ p(rested|tired) = 1/3 & p(rested|rested) = 2/3 \end{array}$$

and then the matrix B:

$$\begin{array}{ll} p(0|tired) = 0 & p(0|rested) = 2/5 \\ p(1|tired) = 1/4 & p(1|rested) = 1/5 \\ p(2|tired) = 1/2 & p(2|rested) = 1/5 \end{array}$$

Slide adapted from David Mortensen

Parameters of an HMM for POS

transition probabilities

$A =$

	N	V	O
N	0.1	0.6	0.3
V	0.3	0.3	0.4
O	0.3	0.4	0.3

emission probabilities

$B =$

	I	m	gonna	make	him	an	offer	he	can	t	refuse
N	0.1	0.00001	0.00001	0.2	0.1	0.00001	0.2	0.1	0.1	0.00001	0.19996
V	0.00001	0.1	0.2	0.2	0.00001	0.00001	0.05	0.00001	0.19995	0.00001	0.25
O	0.00001	0.00001	0.00001	0.00001	0.00001	0.5	0.00001	0.00001	0.00001	0.49991	0.00001

Slide adapted from David Mortensen

Decoding HMMs: Viterbi algorithm

Often, we want to decode HMMs

Input: A trained HMM and a series of observations

Output: A series of labels, corresponding to hidden states of the HMM

This task shows up many times:

- Labeling words according to their parts of speech
- Labeling words according to whether they are at the beginning, otherwise inside of, or outside of a name
- Inferring the sequence of tired and not tired days in the month of your instructor based on his Coke Zero consumption

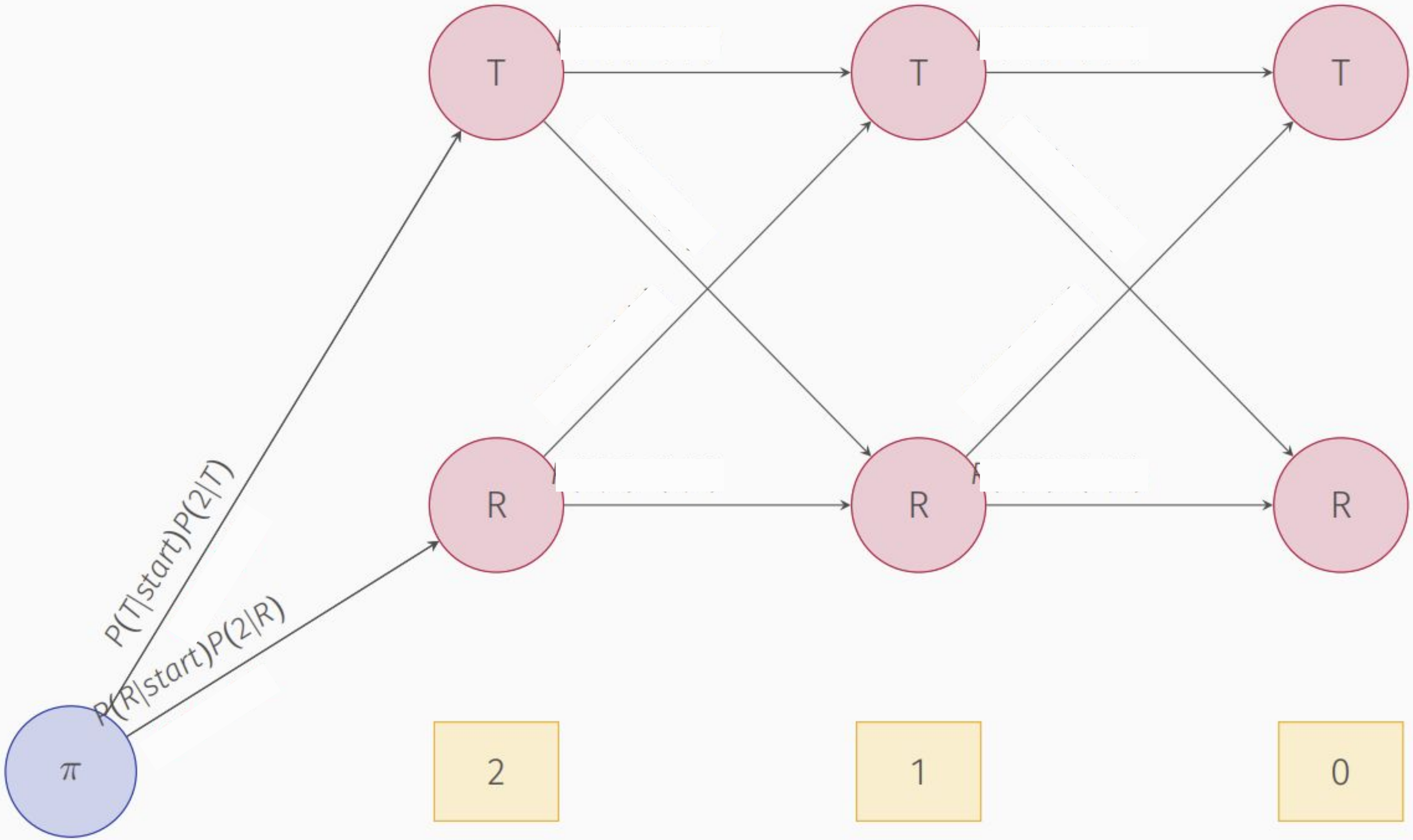
More formally, given as input an HMM $\lambda = (A, B)$ and a sequence of observations $O = o_1, o_2, \ldots, o_T$, find the most probable sequence of states $Q = q_1, q_2, \ldots, q_T$

Slide adapted from David Mortensen

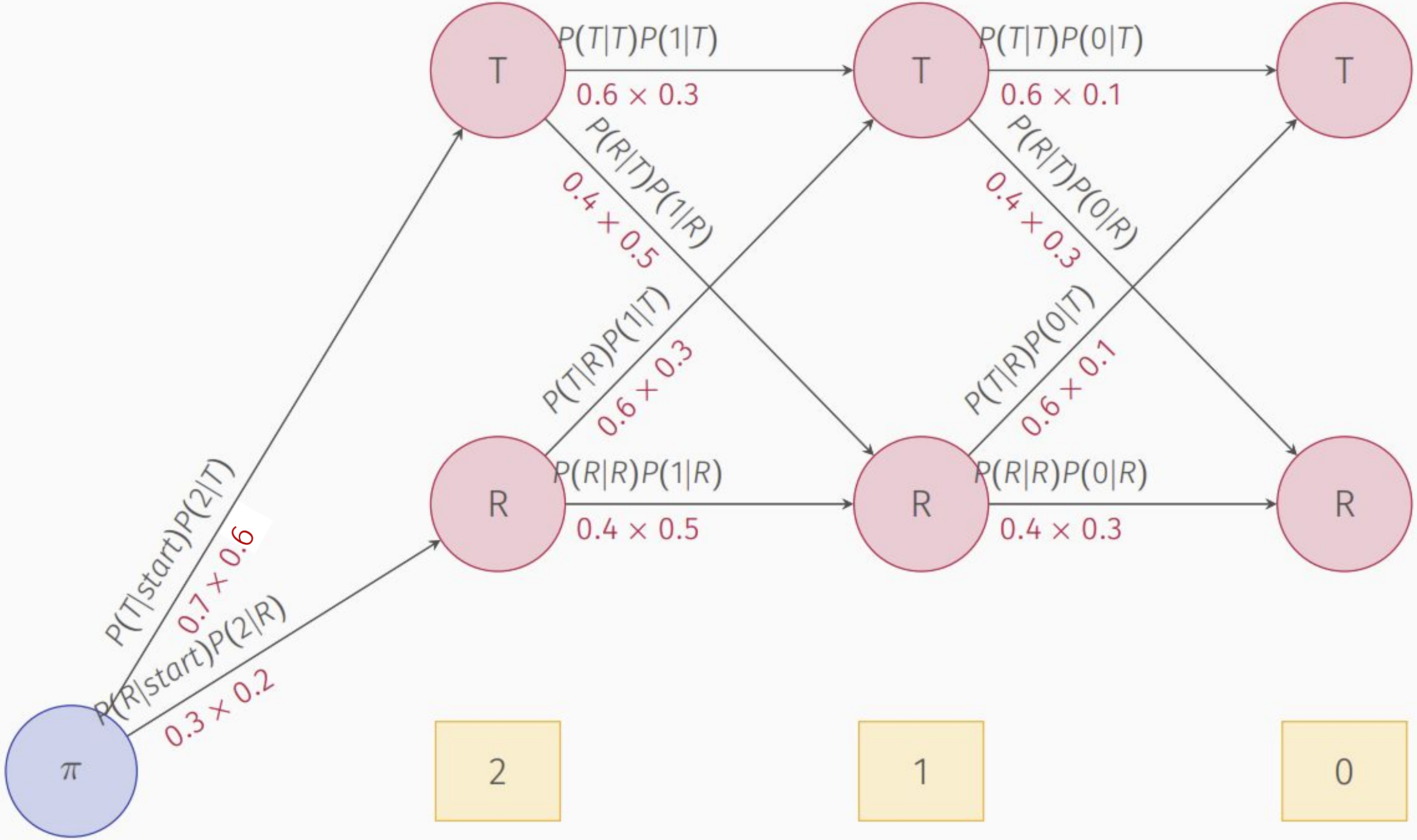
Dynamic programming

- Solves a larger problem by combining solutions to smaller subproblems
- Fills in a table for those subproblems
- Often used in NLP to compute optimal paths through sequences

Computing a Forward Trellis



Computing a Forward Trellis



Slide credit: David Mortensen

Can we do better than the Forward Algorithm for decoding?

- Computing the probability for all possible sequences of states with the forward trellis is computationally infeasible
- The set of possible state sequences (e.g. TTT, TRT, TRR, RRR, ...) grows exponentially as the number of states N grows!

That's where dynamic programming comes in!

- Skip the repeated computation by recording the best probabilities for subsequences along the way
- Viterbi algorithm

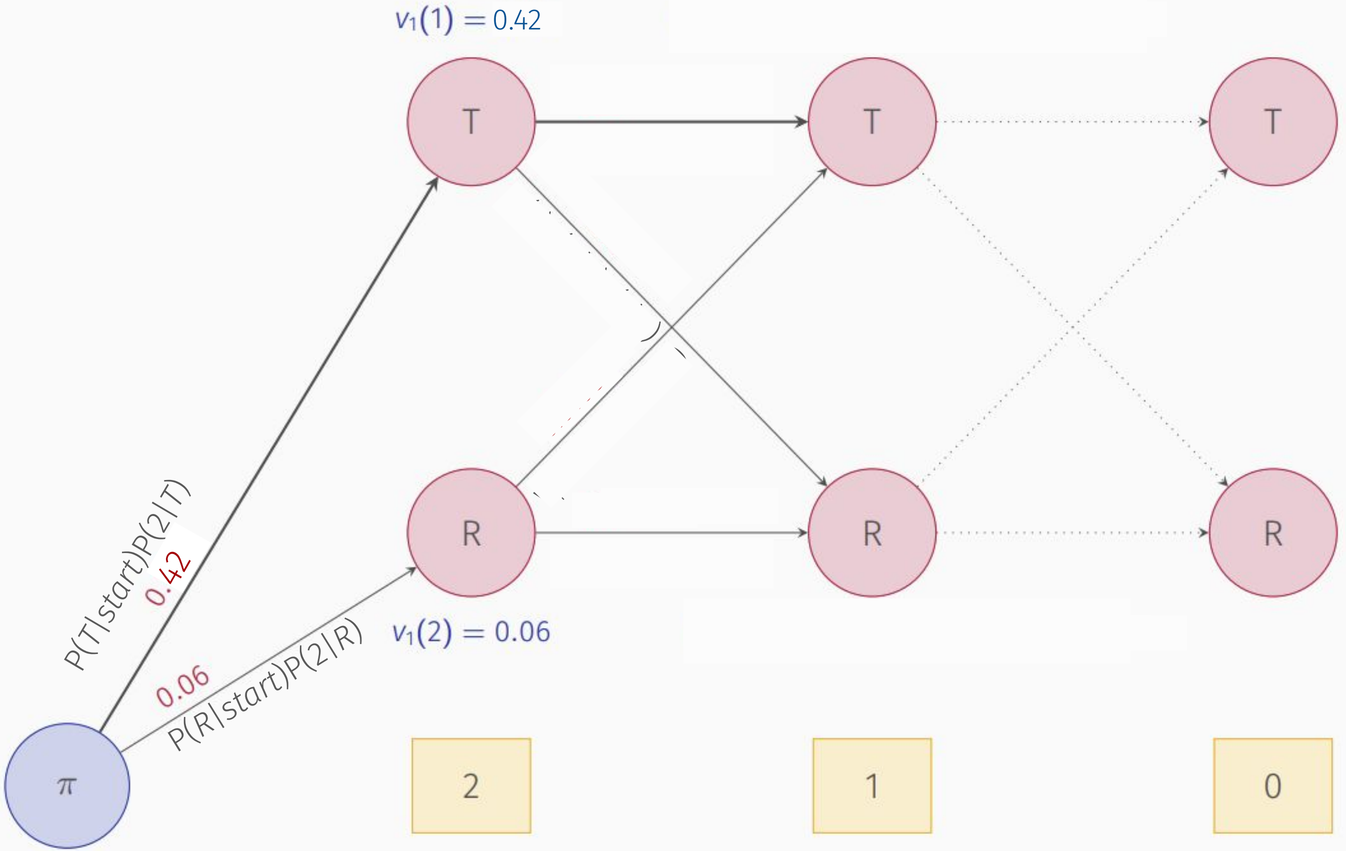
Slide credit: David Mortensen

The Viterbi Algorithm Can Be Used to Decode HMMs

function VITERBI(observations $O = o_1, o_2, \ldots, 0_T$, state-graph of length N)
  $V[N, T] \leftarrow$ empty path probability matrix
  $B[N, T] \leftarrow$ empty backpointer matrix
  for each $s \in 1..N$ **do**
    $V[s, 1] \leftarrow \pi_s \cdot b_s(o_1)$
    $B[s, 1] \leftarrow 0$
  for each $t \in 2..T$ **do**
    for each $s \in 1..N$ **do**
      $V[s, t] \leftarrow \max_{s'=1}^{N} V[s', t-1] \cdot a_{s',s} \cdot b_s(o_t)$
      $B[s, t] \leftarrow \operatorname{argmax}_{s'=1}^{N} V[s', t-1] \cdot a_{s',s} \cdot b_s(o_t)$
  bestpathprob $\leftarrow \max_{s=1}^{N} V[s, T]$
  bestpathpointer $\leftarrow \max_{s=1}^{N} V[s, T]$
  bestpath $\leftarrow$ path starting at *bestpathpointer* that follows b to states back in time.
  return *bestpath*, *bestpathprob*

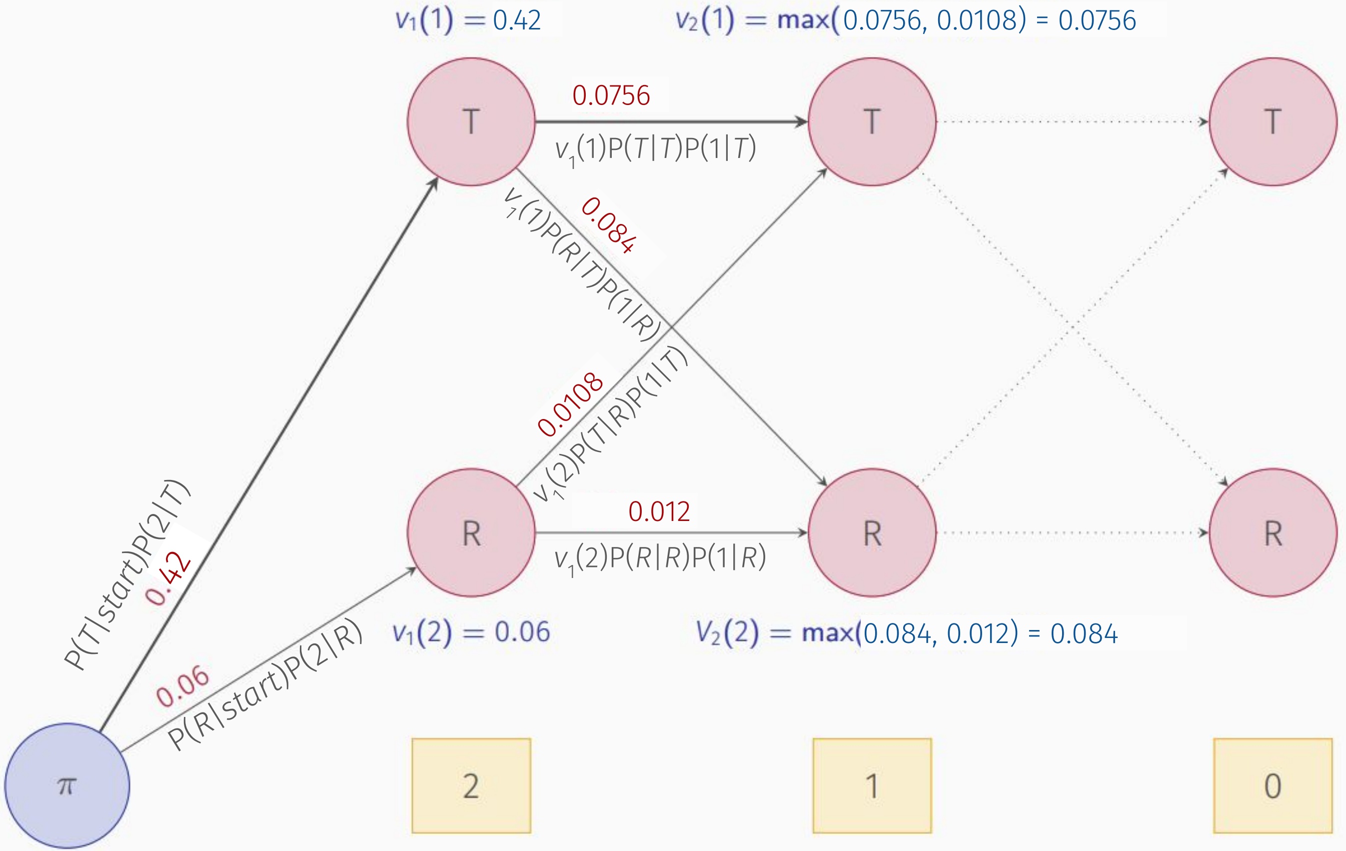
Slide credit: David Mortensen

Using Viterbi to Decode an HMM



Slide adapted from David Mortensen

Using Viterbi to Decode an HMM



Slide adapted from David Mortensen

	B	I	O
B	0	0.5	0.5
I	.1	0	0.9
O	0.2	0	0.8

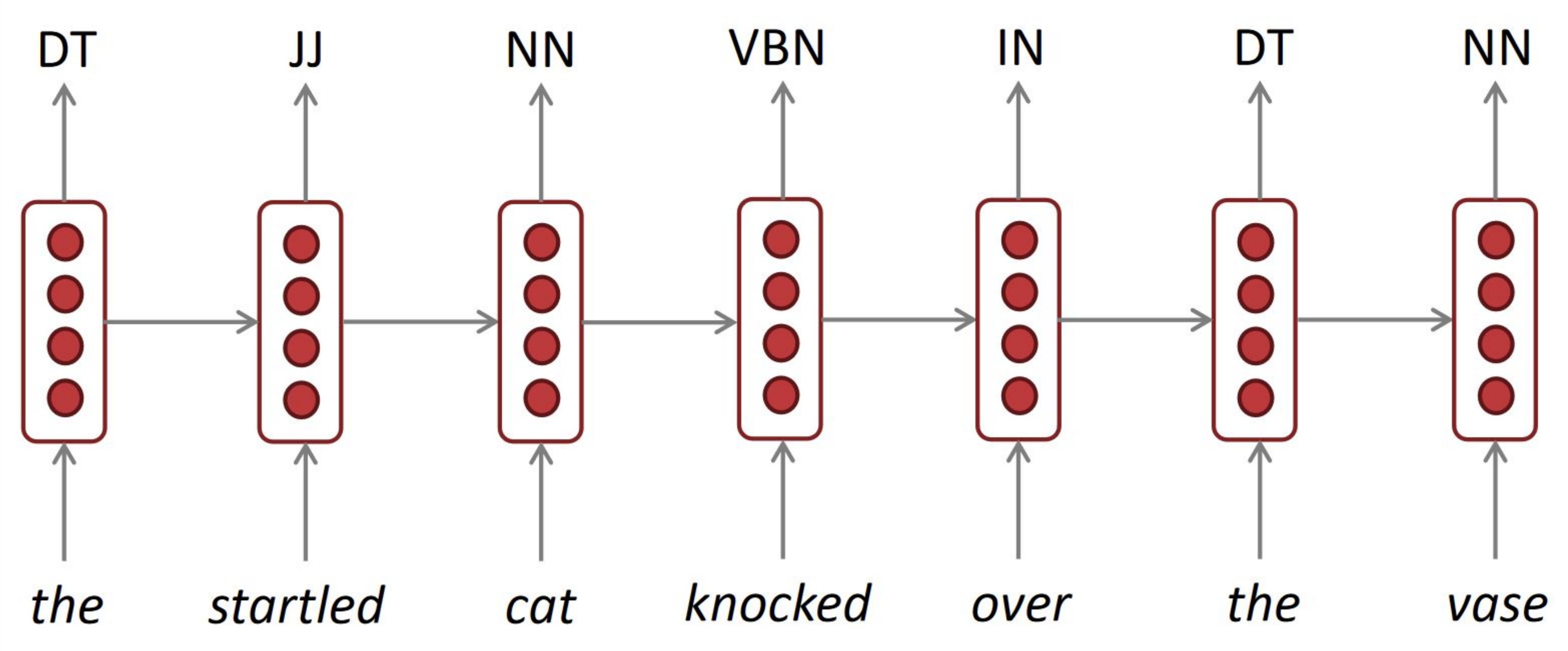
	United	States	live	in
B	0.8	0.3	0	0
I	0.1	0.6	0.1	0.1
O	0.1	0.1	0.9	0.9

	π
B	0.2
I	0
O	0.8

To decode:
live in United States

Neural sequence labeling

RNNs can be used for sequence labeling



Slide adapted from Chris Manning

BERT can be used for sequence labeling

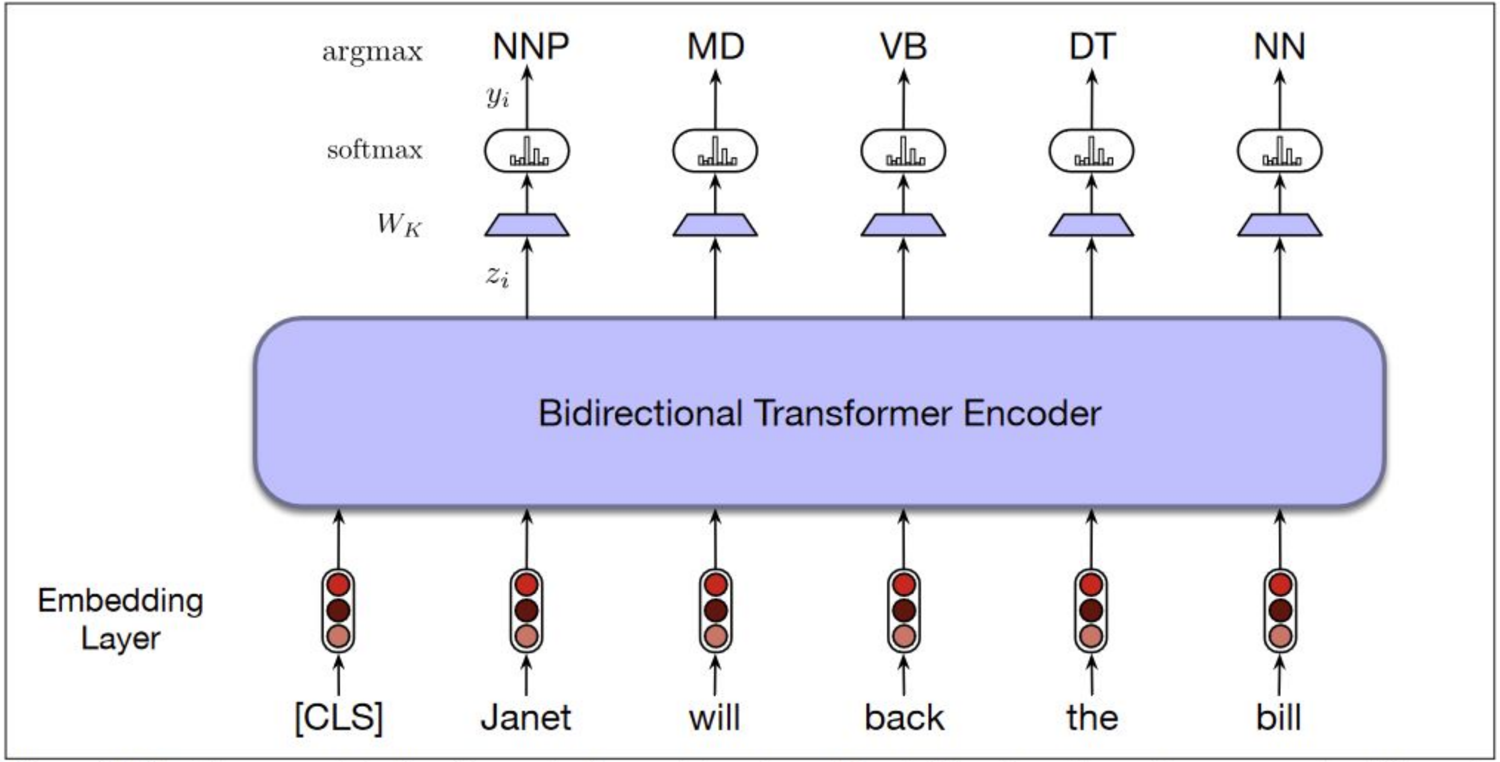


Figure 11.9 Sequence labeling for part-of-speech tagging with a bidirectional transformer encoder. The output vector for each input token is passed to a simple k-way classifier.

Slide adapted from Jurafsky & Martin

An alternative to BIO: span-based NER

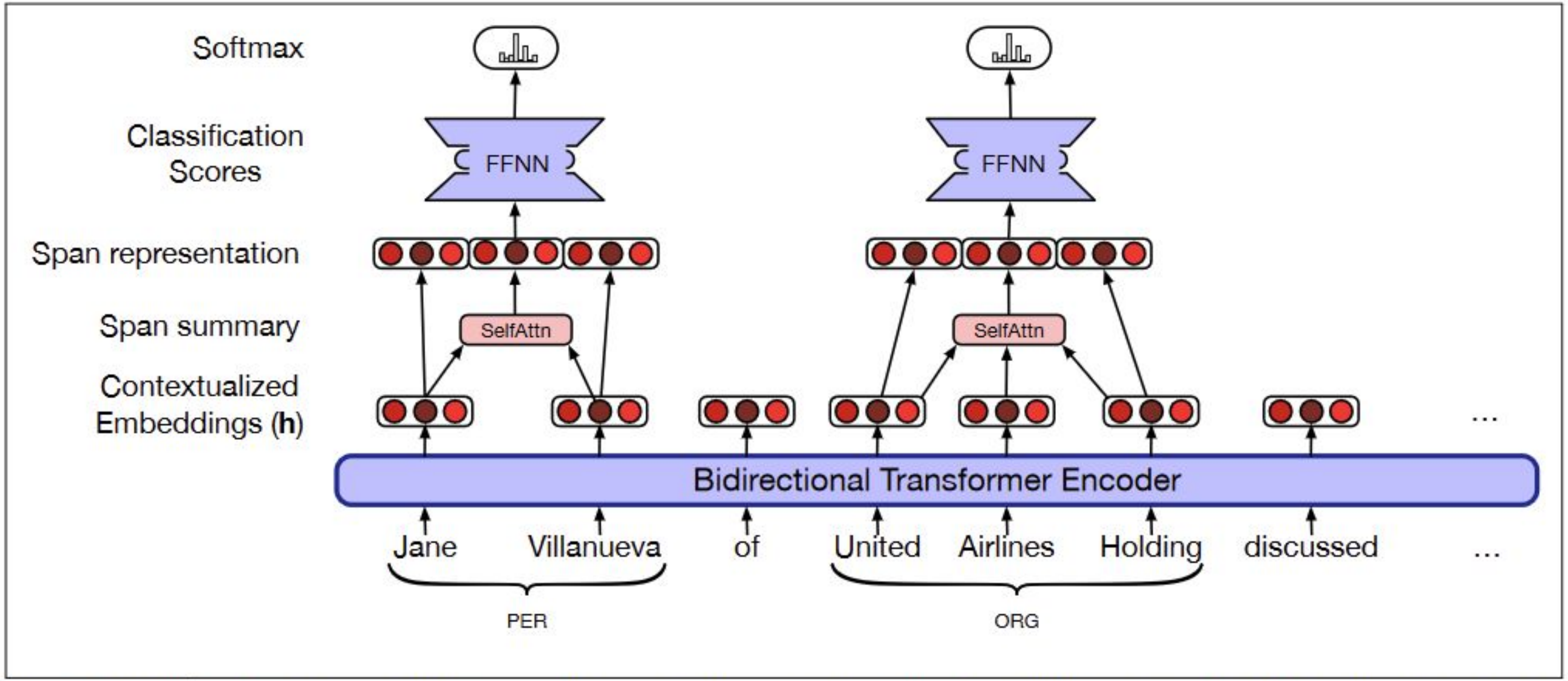


Figure 11.10 A span-oriented approach to named entity classification. The figure only illustrates the computation for 2 spans corresponding to ground truth named entities. In reality, the network scores all of the $\frac{T(T-1)}{2}$ spans in the text. That is, all the unigrams, bigrams, trigrams, etc. up to the length limit.

Slide adapted from Jurafsky & Martin

Wrapping up

- If enough annotated training data is available, HMMs can be trained with MLE
- The Viterbi algorithm is used for decoding HMMs
- RNNs and transformers can be trained to do sequence labeling

Questions?