CS 2731 / ISSP 2230
Introduction to Natural Language Processing

Session 9: Feedforward neural networks

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Course logistics

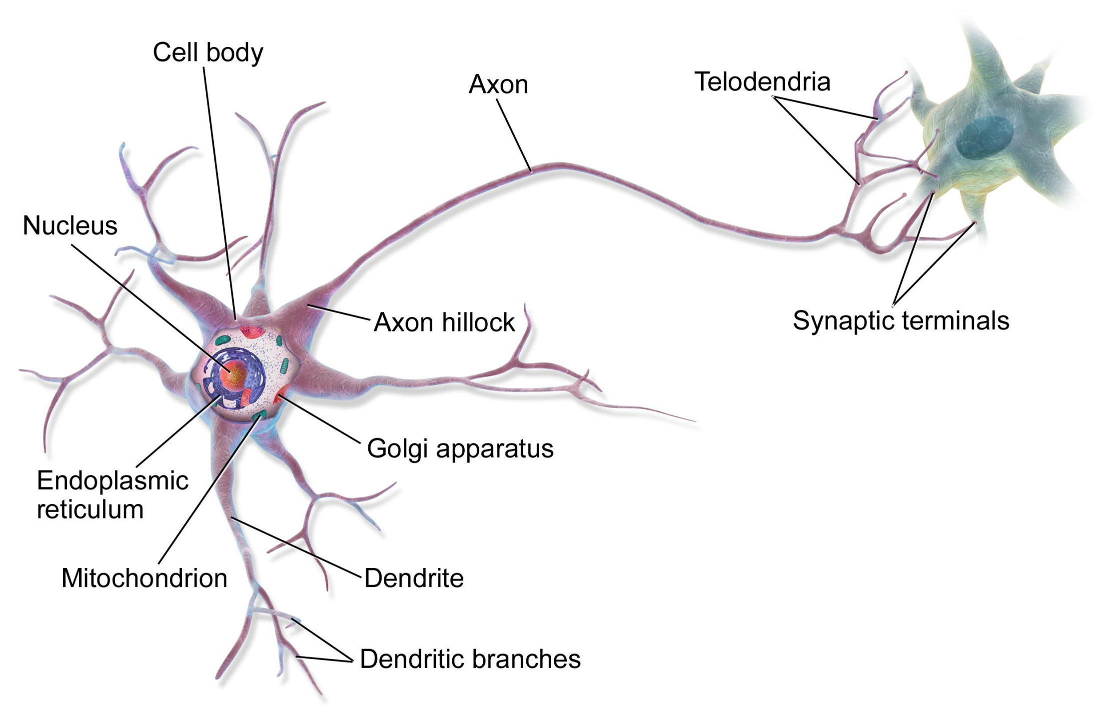
- Homework 2 is due **Thu Feb 15**
  - Text classification
  - Written and programming components
  - Optional Kaggle competition for best LR and NN politeness classifiers
- Projects
  - Proposal and literature review is **due Thu Feb 22**
    - Instructions are on the project webpage
  - It's good to start the literature review early
  - Look for NLP papers in ACL Anthology, Semantic Scholar, and Google Scholar

Lecture overview: feedforward neural networks

- Neural network fundamentals
- Non-linear activation functions
- Linear algebra review
- Feedforward neural networks as classifiers
- Training feedforward neural networks (backpropagation)

Neural network fundamentals

This is in your brain



Neural Network Unit: This is not in your brain

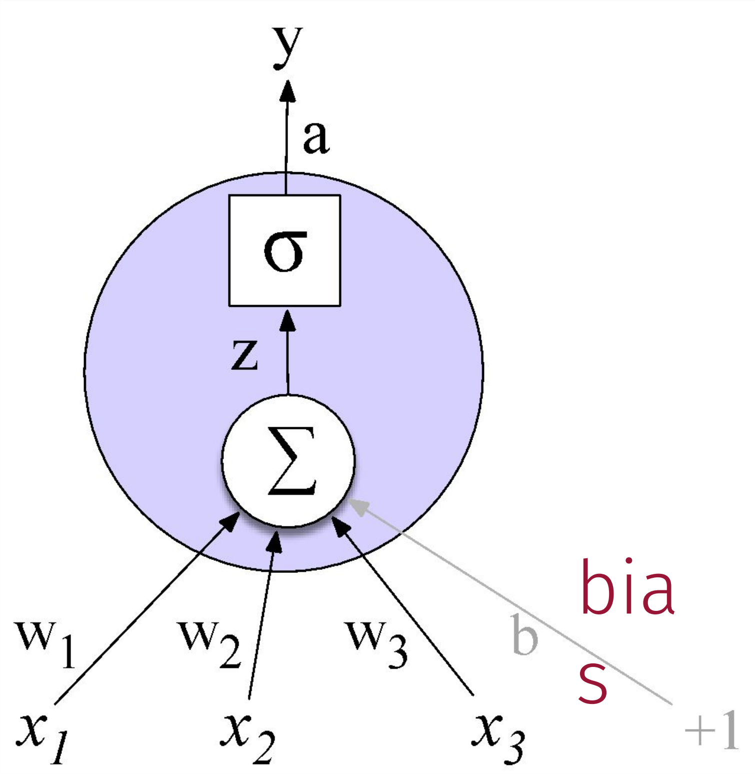
Output value

Non-linear transform

Weighted sum

Weights

Input layer



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The Variables in Our Very Important Formula

$\mathbf{x}$ A vector of features of n dimensions (like number of positive sentiment words, length of document, etc.)

$\mathbf{w}$ A vector of weights of n dimensions specifying how discriminative each feature is

b A scalar bias term that shifts z

z The raw score

y A random variable (e.g., $y = 1$ means positive sentiment and $y = 0$ means negative sentiment

Slide adapted from David Mortensen

The fundamental equation that describes a unit of a neural network should look very familiar:

$$z = b + \sum_i w_i x_i \tag{1}$$

Which we will represent as

$$z = \mathbf{w} \cdot \mathbf{x} + b \tag{2}$$

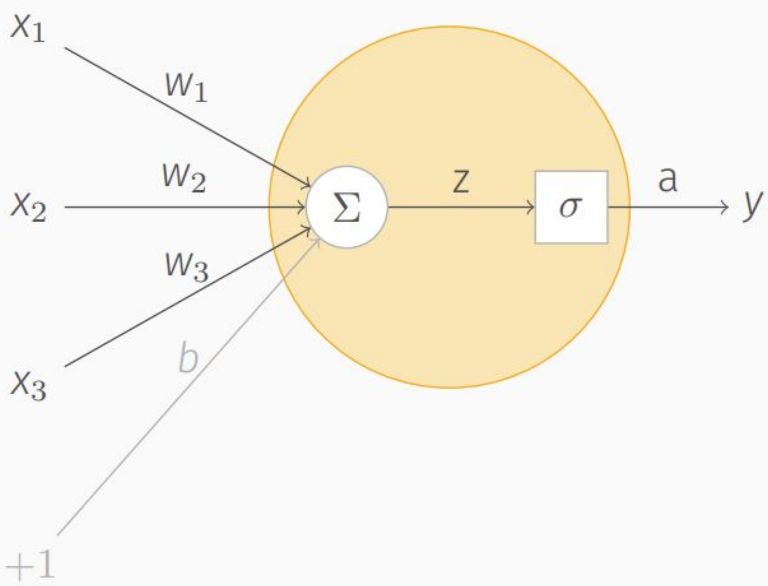
But we do not use z directly. Instead, we pass it through a non-linear function, like the sigmoid function:

$$y = \sigma(z) = \frac{1}{1 + e^{-z}} \tag{3}$$

(which has some nice properties even though, in practice, we will prefer other functions like tanh and ReLU).

Slide adapted from David Mortensen

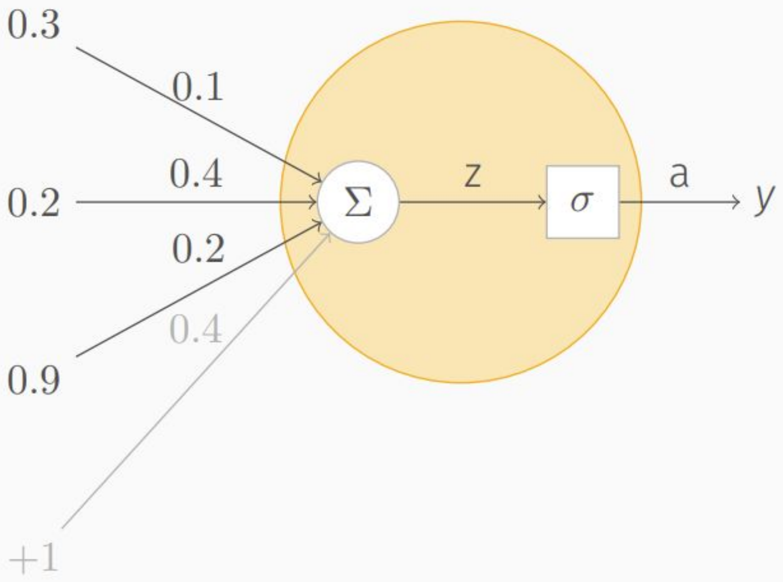
A Unit Illustrated



Take, for example, a scenario in which our unit has the weights [0.1, 0.4, 0.2] and the bias term 0.4 and the input vector x has the values [0.3, 0.2, 0.9].

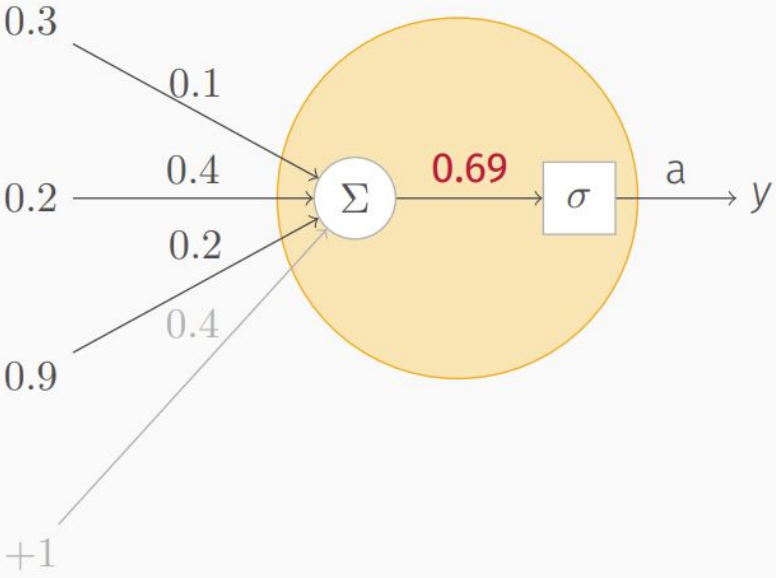
Slide adapted from David Mortensen

Filling in the Input Values and Weights



Slide adapted from David Mortensen

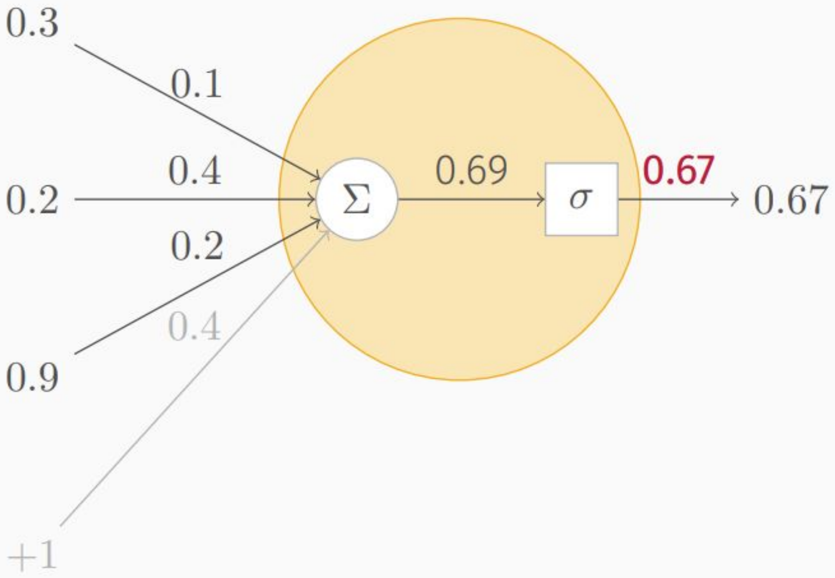
Multiplying the Input Values and Weights and Summing Them (with the Bias Term)



$$z = x_1 w_1 + x_2 w_2 + x_3 w_3 + b = 0.1(0.3) + 0.4(0.2) + 0.2(0.9) + 0.4 = 0.69 \quad (4)$$

Slide adapted from David Mortensen

Applying the Activation Function (Sigmoid)



$$y = \sigma(0.69) = \frac{1}{1 + e^{-0.69}} = 0.67 \tag{5}$$

Slide adapted from David Mortensen

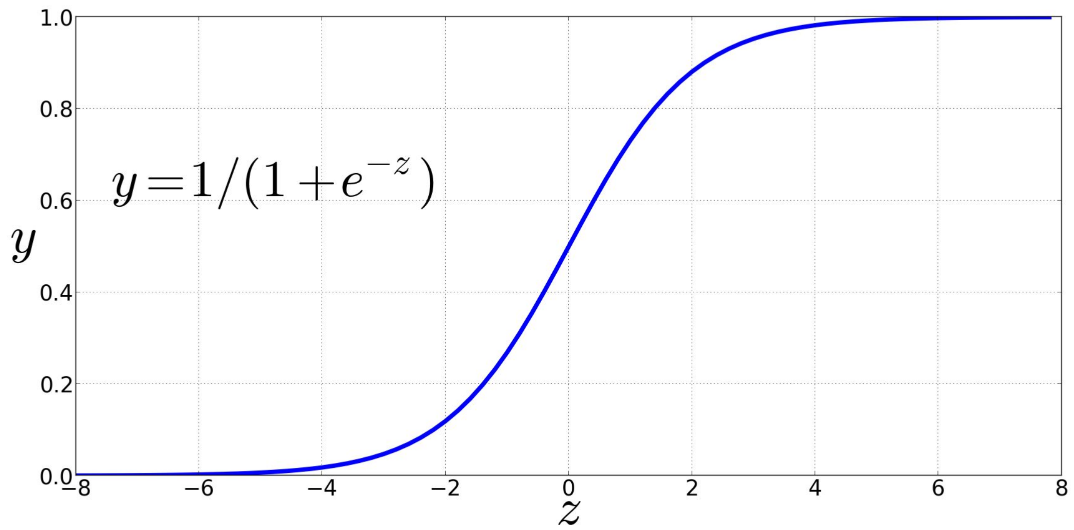
Non-linear activation functions

Non-Linear Activation Functions

We're already seen the sigmoid for logistic regression:

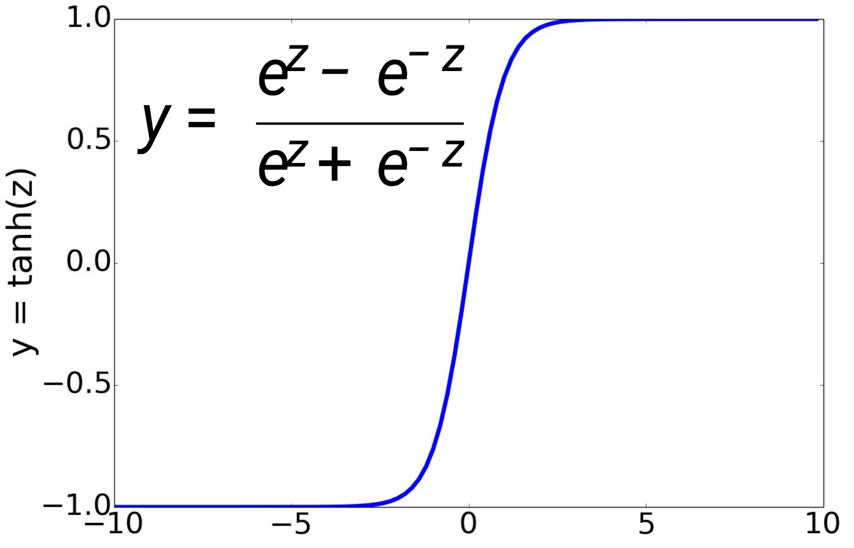
Sigmoid

$$y = s(z) = \frac{1}{1 + e^{-z}}$$



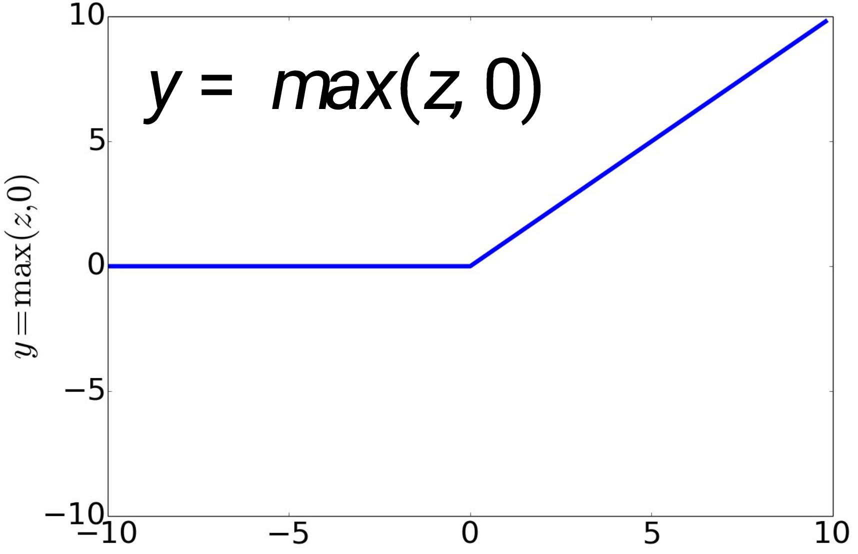
Non-Linear Activation Functions besides sigmoid

Most Common:



$$y = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

tanh



$$y = max(z, 0)$$

ReLU

Rectified Linear Unit

Slide adapted from Jurafsky & Martin

A little linear algebra

So Far, We Have Assume You Know Dot Products

$$\boldsymbol{a} = (a_1, a_2, a_3)$$

$$\boldsymbol{b} = (b_1, b_2, b_3)$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Slide adapted from David Mortensen

Now, You Need to Multiply Matrices

A matrix is an array of numbers

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

Two rows, three columns.

Slide adapted from David Mortensen

It’s Easy to Multiple a Matrix by a Scalar

$$2 \cdot \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} \color{red}{2} \cdot 5 & \color{red}{2} \cdot 2 \\ \color{red}{2} \cdot 3 & \color{red}{2} \cdot 1 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 2 & 4 \end{bmatrix}$$

Slide adapted from David Mortensen

Multiplying Matrices by Matrices Is Slightly Trickier

Let a_1 and a_2 be the row vectors of matrix A and b_1 and b_2 be the column vectors of a matrix B. Find $C = AB$

$$\begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} a_1 \cdot b_1 & a_1 \cdot b_2 \\ a_2 \cdot b_1 & a_2 \cdot b_2 \end{bmatrix} = \begin{bmatrix} 38 & 17 \\ 26 & 14 \end{bmatrix}$$

A must have the same number of rows as B has columns.

Slide adapted from David Mortensen

Multiplying a Matrix by a Vector Is Roughly the Same

Multiplying a matrix by a vector is like multiply a matrix by a matrix with one column:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{bmatrix}$$

The result is a vector.

Slide adapted from David Mortensen

Matrix multiplication is not hard but inference with neural nets is mostly this (plus some non-linear functions)

Slide adapted from David Mortensen

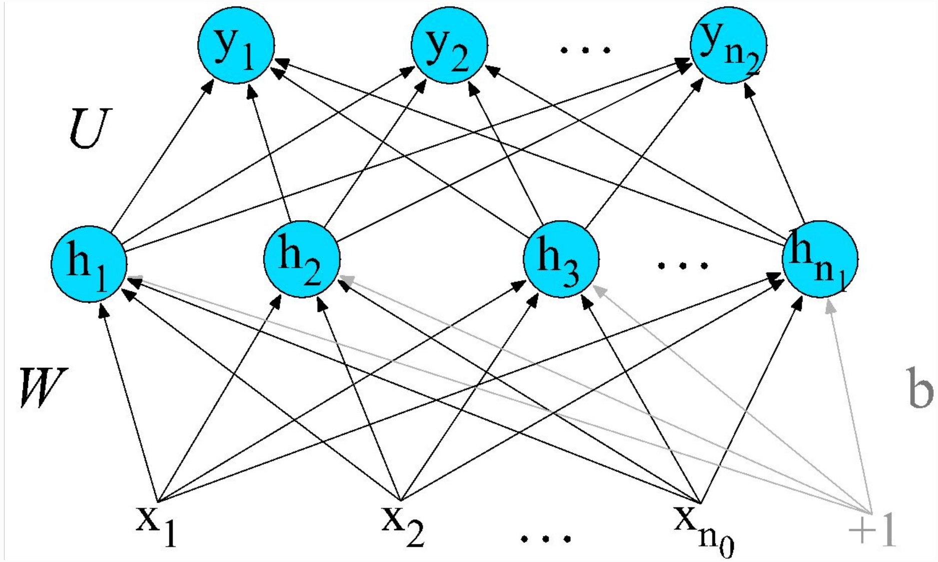
Feedforward neural networks

Adding multiple units to a neural network increases its power to learn patterns in data. **Feedforward Neural Nets (FFNNs or MLPs)**

Slide adapted from David Mortensen

Feedforward Neural Networks

Can also be called **multi-layer perceptrons** (or **MLPs**) for historical reasons



The simplest FFNN is just binary logistic regression
(INPUT LAYER = feature vector)

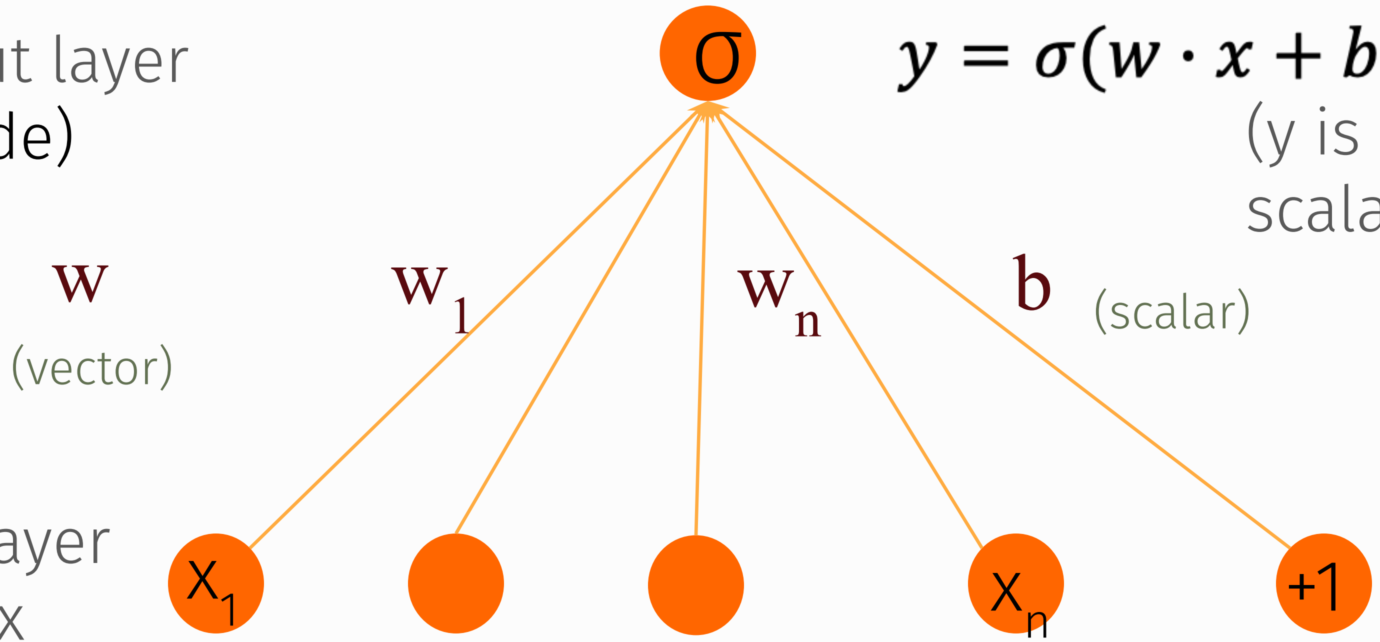
Binary Logistic Regression as a 1-layer Network

(we don't count the input layer in counting layers!)

Output layer
(σ node)

$$y = \sigma(w \cdot x + b)$$

(y is a scalar)



Input layer
vector x

Slide adapted from Jurafsky & Martin

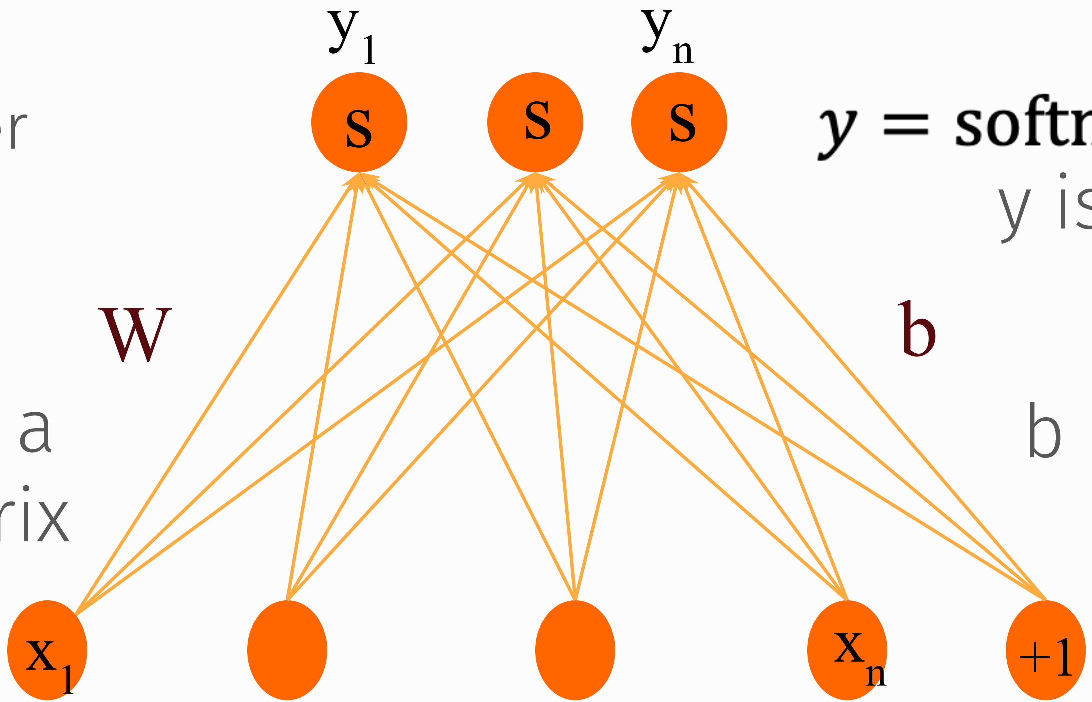
Multinomial Logistic Regression as a 1-layer Network

Fully connected single layer network

Output layer (softmax nodes)

$$y = \mathrm{softmax}(Wx + b)$$

y is a vector



W is a matrix

b is a vector

Input layer scalars

Slide adapted from Jurafsky & Martin

Softmax is a Generalization of Sigmoid

Softmax will show up multiple times in this class as a way of converting numbers into probablities. For a vector z of dimensionality k, the softmax is:

$$\text{softmax}(\mathbf{z}) = \left[\frac{\exp(z_1)}{\sum_{i=1}^{k} exp(z_i)}, \frac{\exp(z_2)}{\sum_{i=1}^{k} exp(z_i)}, \ldots, \frac{\exp(z_n)}{\sum_{i=1}^{k} exp(z_i)}\right] \tag{6}$$

$$\text{softmax}(z_i) = \frac{\exp(z_1)}{\sum_{j=1}^{k} exp(z_j)} \; 1 \leq i \leq k \tag{7}$$

For example, if $z = [0.6, 1.1, -1.5, 1.2, 3.2, -1.1]$ then
$\text{softmax}(x) = [0.055, 0.090, 0.006, 0.099, 0.74, 0.010]$

Slide adapted from David Mortensen

Probability distribution: a statistical function describing all the possible values/probabilities for a random variable within a given range.

Slide adapted from David Mortensen

The real power comes when multiple layers are added

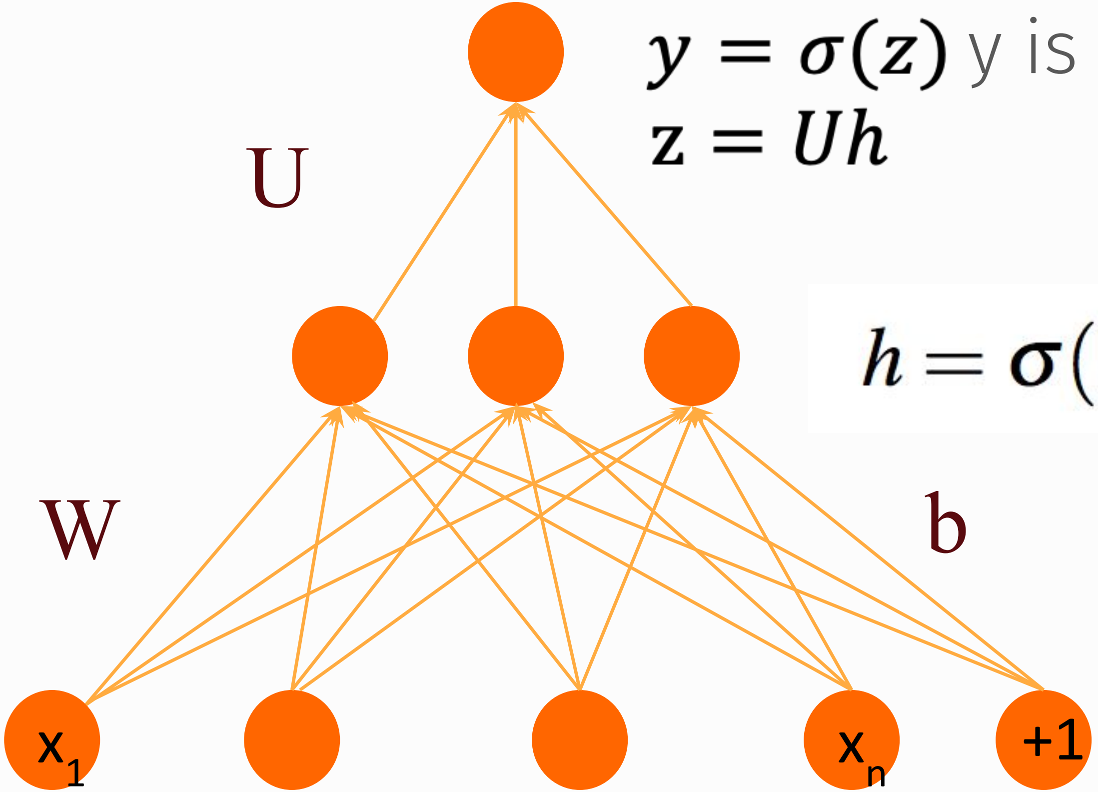
Slide adapted from David Mortensen

Two-Layer Network with scalar output

Output layer
(σ node)

hidden units
(σ node)

Input layer
(vector)



$y = \sigma(z)$ y is a scalar

$z = Uh$

$$h = \sigma(Wx + b)$$

Could be
ReLU
Or tanh

Slide adapted from Jurafsky & Martin

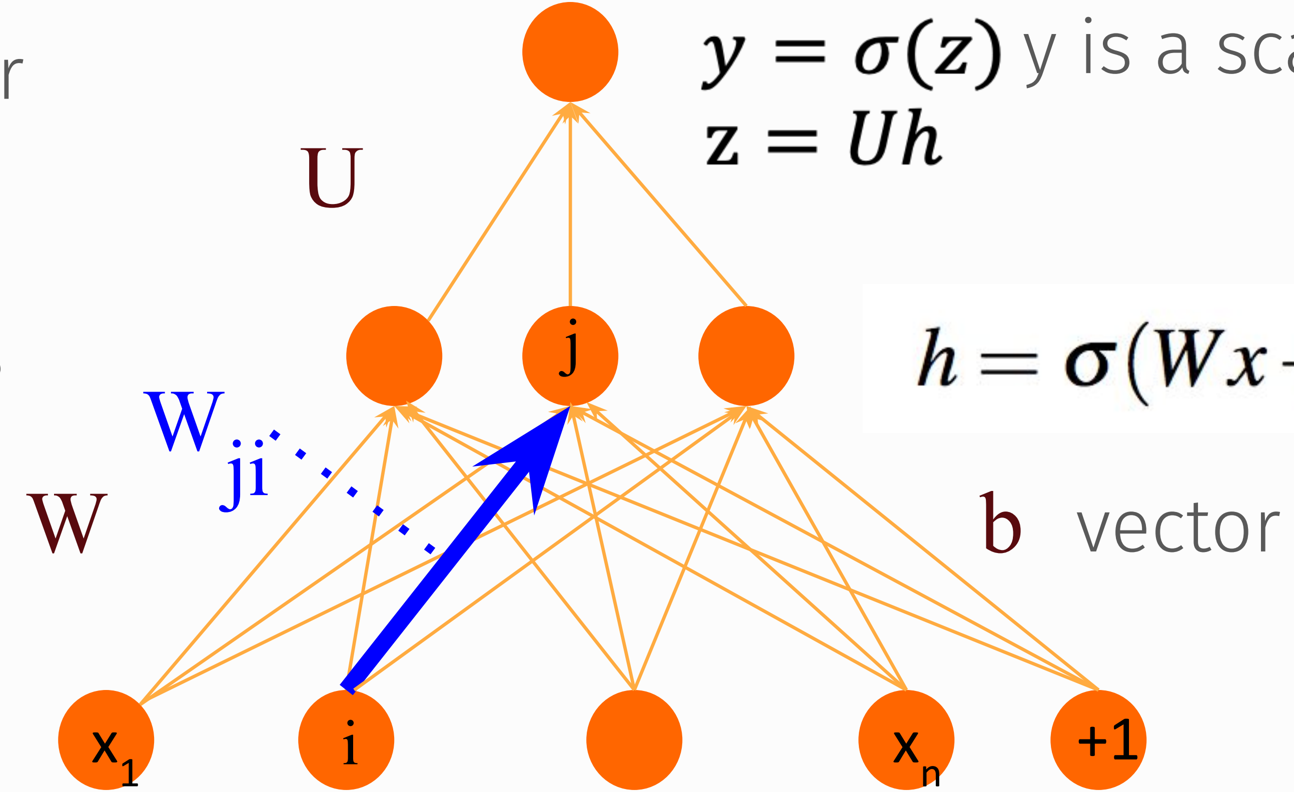
Two-Layer Network with scalar output

Output layer
(σ node)

$y = \sigma(z)$ y is a scalar
$z = Uh$

hidden units
(σ node)

$h = \sigma(Wx + b)$



Input layer
(vector)

Two-Layer Network with scalar output

Output layer
(σ node)

$y = \sigma(z)$ y is a scalar

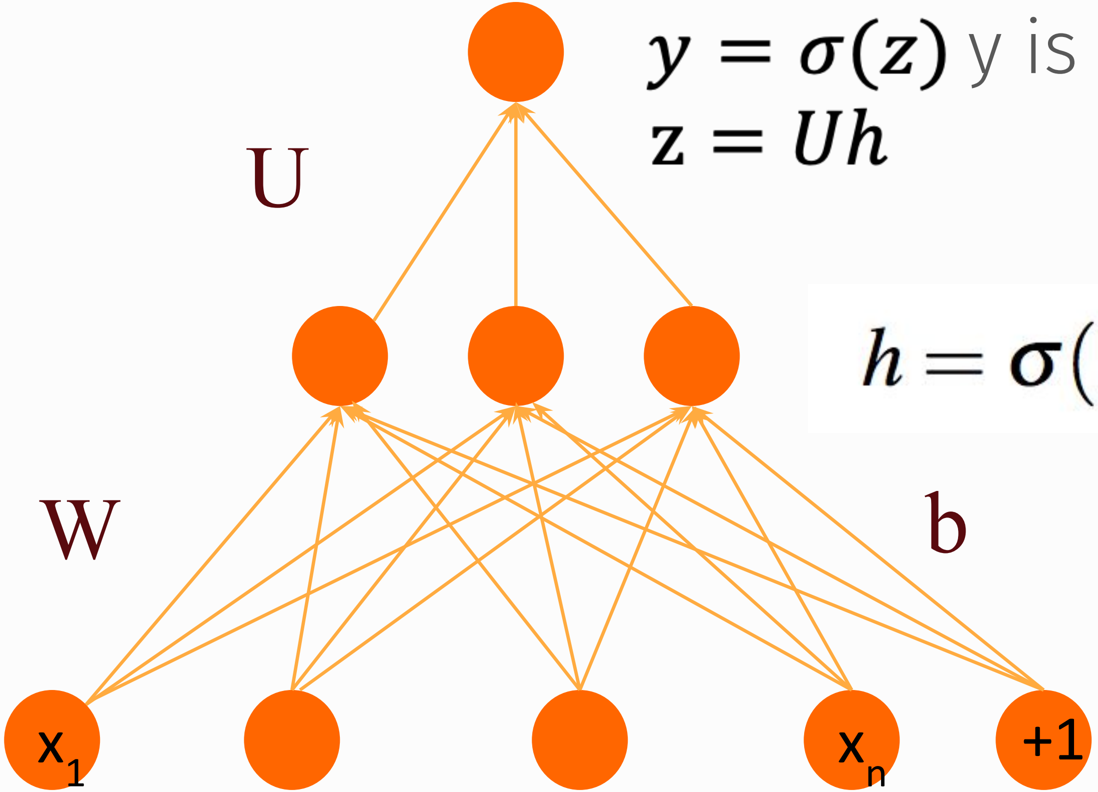
$z = Uh$

hidden units
(σ node)

$h = \sigma(Wx + b)$

Could be
ReLU
Or tanh

Input layer
(vector)



Slide adapted from Jurafsky & Martin

Two-Layer Network with softmax output

Output layer
(σ node)

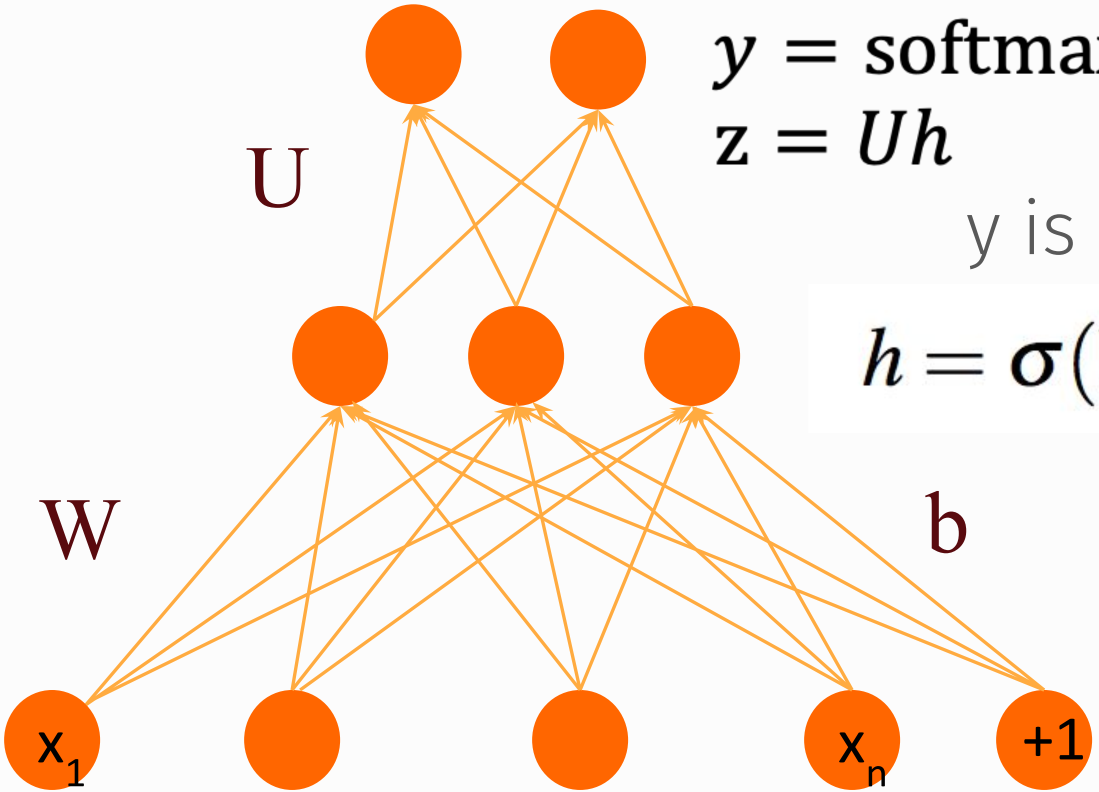
$$y = \text{softmax}(z)$$
$$z = Uh$$

y is a vector

hidden units
(σ node)

$$h = \sigma(Wx + b)$$

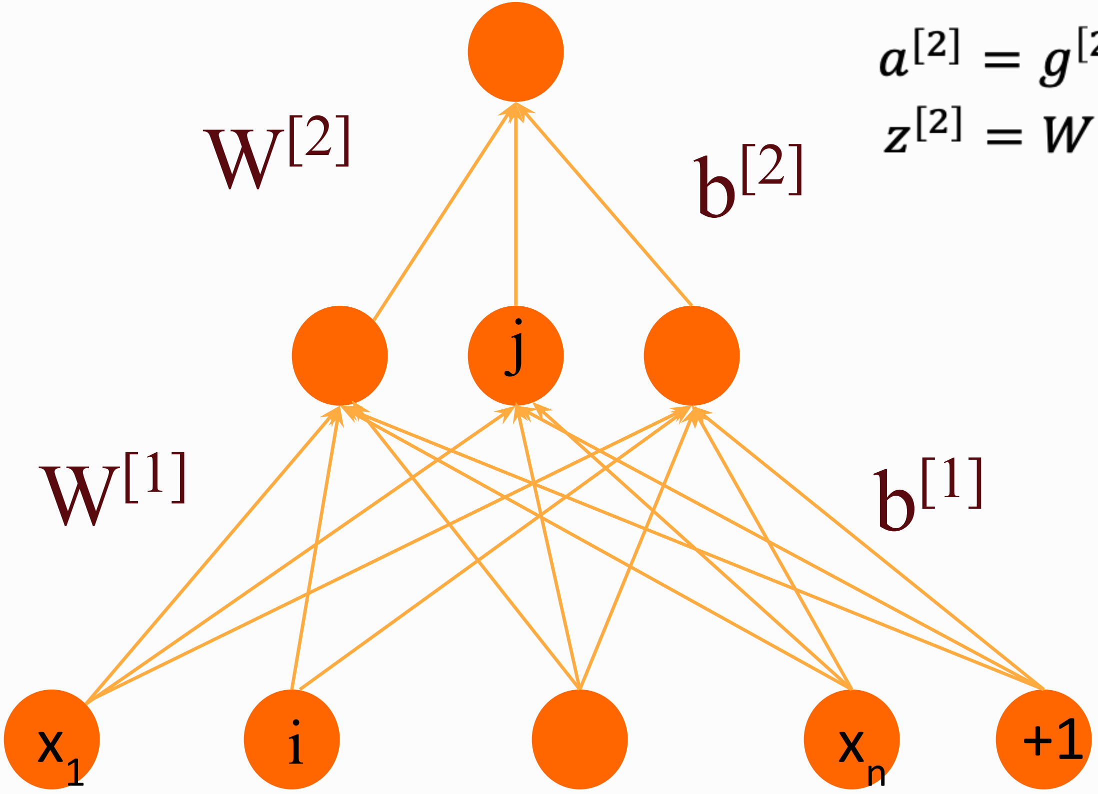
Could be
ReLU
Or tanh



Input layer
(vector)

Slide adapted from Jurafsky & Martin

Multi-layer Notation



$$y = a^{[2]}$$

$$a^{[2]} = g^{[2]}(z^{[2]})$$ sigmoid or softmax

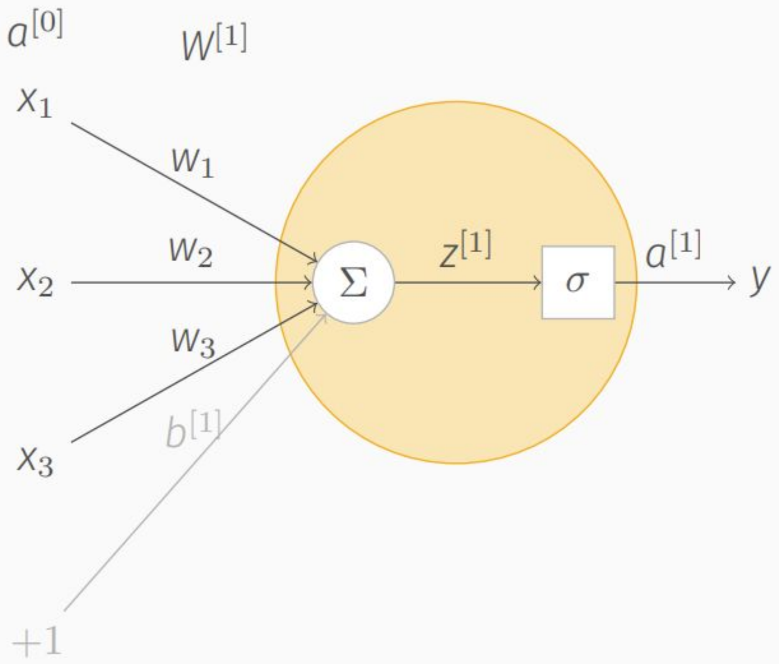
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$ ReLU

$$z^{[1]} = W^{[1]}a^{[0]} + b^{[1]}$$

$$a^{[0]}$$

A Forward Pass in Terms of Multi-Layer Notation



for each $i \in 1..n$ **do**

$\quad z^{[i]} \leftarrow W^{[i]} a^{[i-1]} + b^{[i]}$

$\quad a^{[i]} \leftarrow g^{[i]}(z^{[i]})$

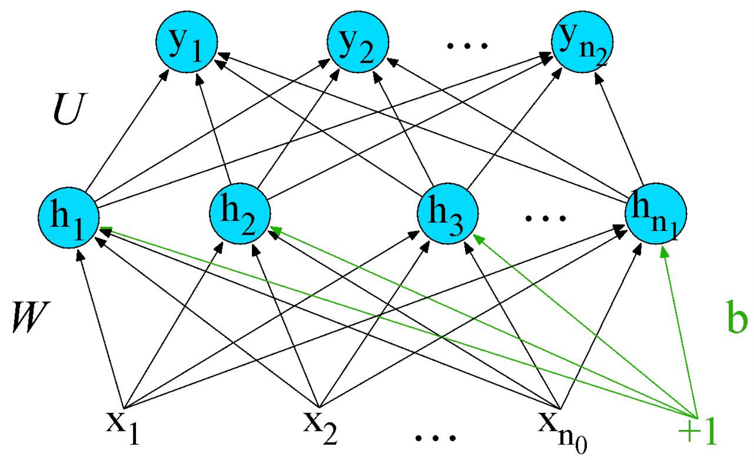
end for

$\hat{y} \leftarrow a^{[n]}$

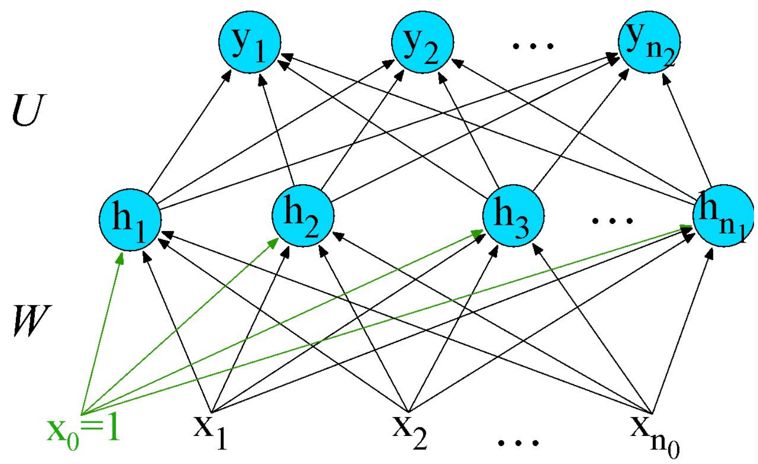
Slide credit: David Mortensen

Replacing the bias unit

Instead of:



We'll do this:



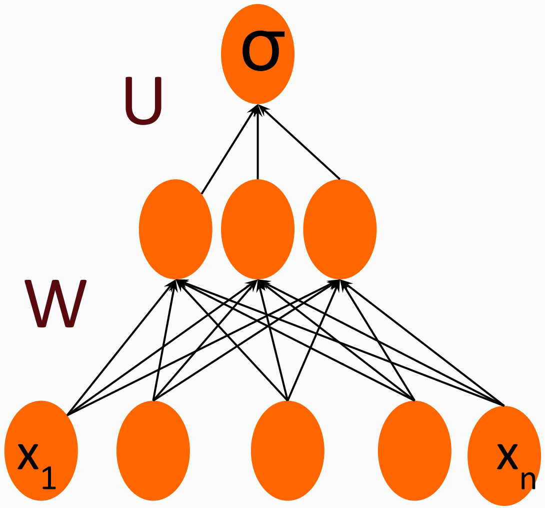
Feedforward neural nets as classifiers

Classification: Sentiment Analysis

We could do exactly what we did with logistic regression

Input layer are binary features as before

Output layer is 0 or 1



Slide adapted from Jurafsky & Martin

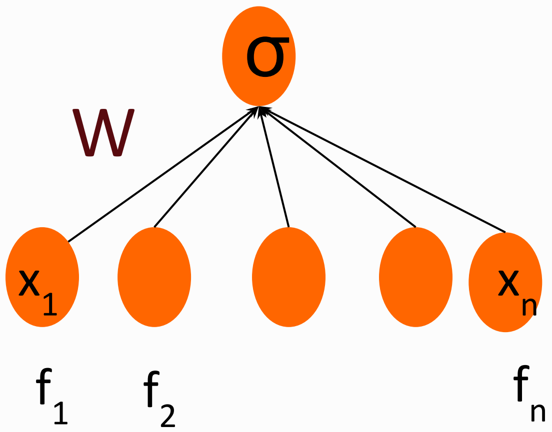
Sentiment Features

Var	Definition
x_1	count(positive lexicon) $\in$ doc)
x_2	count(negative lexicon) $\in$ doc)
x_3	$\begin{cases} 1 & \text{if ``no''} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$
x_4	count(1st and 2nd pronouns $\in$ doc)
x_5	$\begin{cases} 1 & \text{if ``!''} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$
x_6	log(word count of doc)

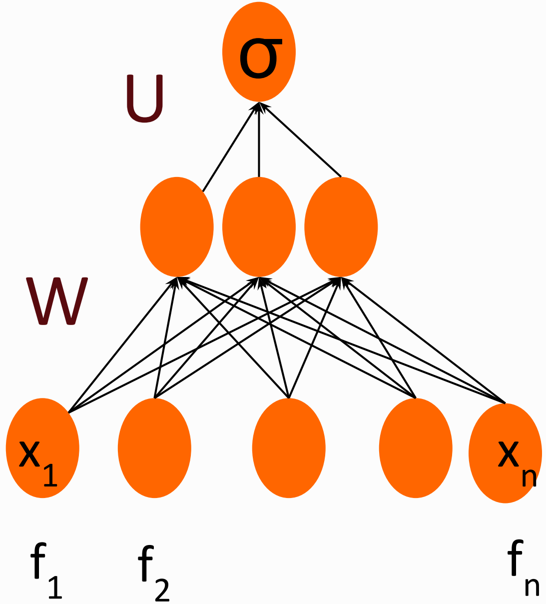
Slide adapted from Jurafsky & Martin

Feedforward nets for simple classification

Logistic Regression



2-layer feedforward network



Just adding a hidden layer to logistic regression

- allows the network to use non-linear interactions between features
- which may (or may not) improve performance.

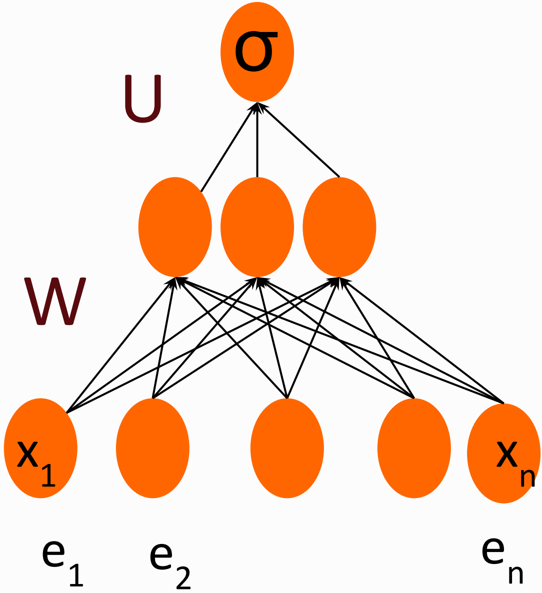
Slide adapted from Jurafsky & Martin

Even better: representation learning

The real power of deep learning comes from the ability to **learn** features from the data

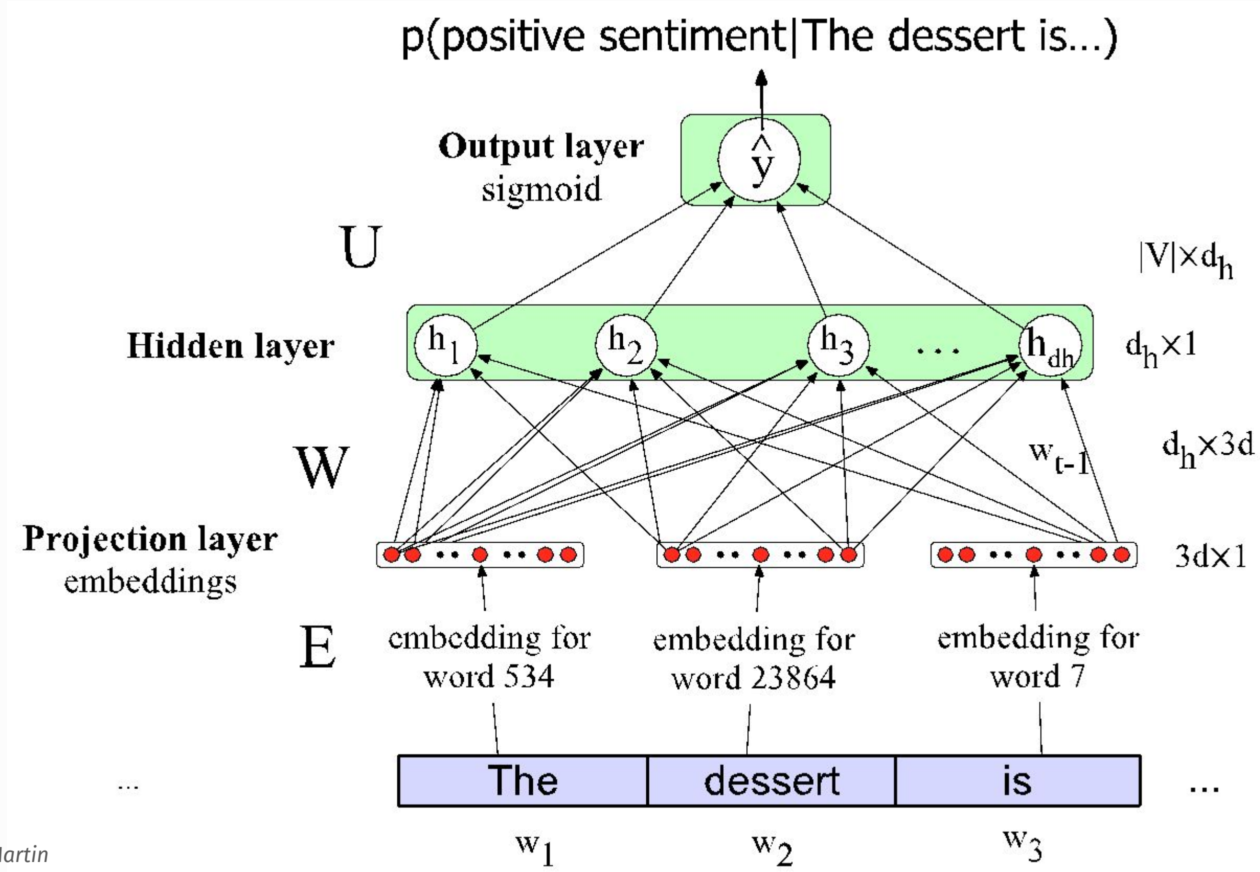
Instead of using hand-built human-engineered features for classification

Use learned representations like embeddings!



Slide adapted from Jurafsky & Martin

Neural net classification with embeddings as input features!



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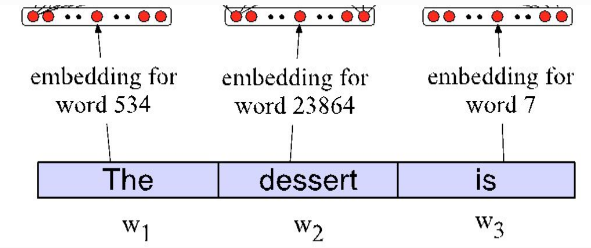
Issue: texts come in different sizes



This assumes a fixed size length (3)!
Kind of unrealistic.
Some simple solutions (more sophisticated solutions later)

1. Make the input the length of the longest review
   - If shorter then pad with zero embeddings
   - Truncate if you get longer reviews at test time
2. Create a single "sentence embedding" (the same dimensionality as a word) to represent all the words
   - Take the mean of all the word embeddings
   - Take the element-wise max of all the word embeddings
     - For each dimension, pick the max value from all words

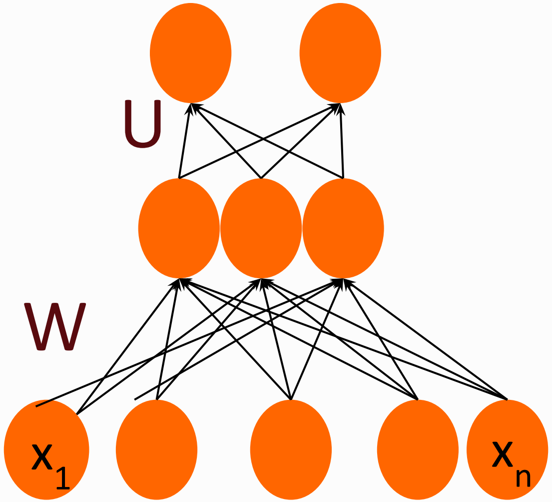
Slide adapted from Jurafsky & Martin

Reminder: Multiclass Outputs

What if you have more than two output classes?

- Add more output units (one for each class)
- And use a "softmax layer"

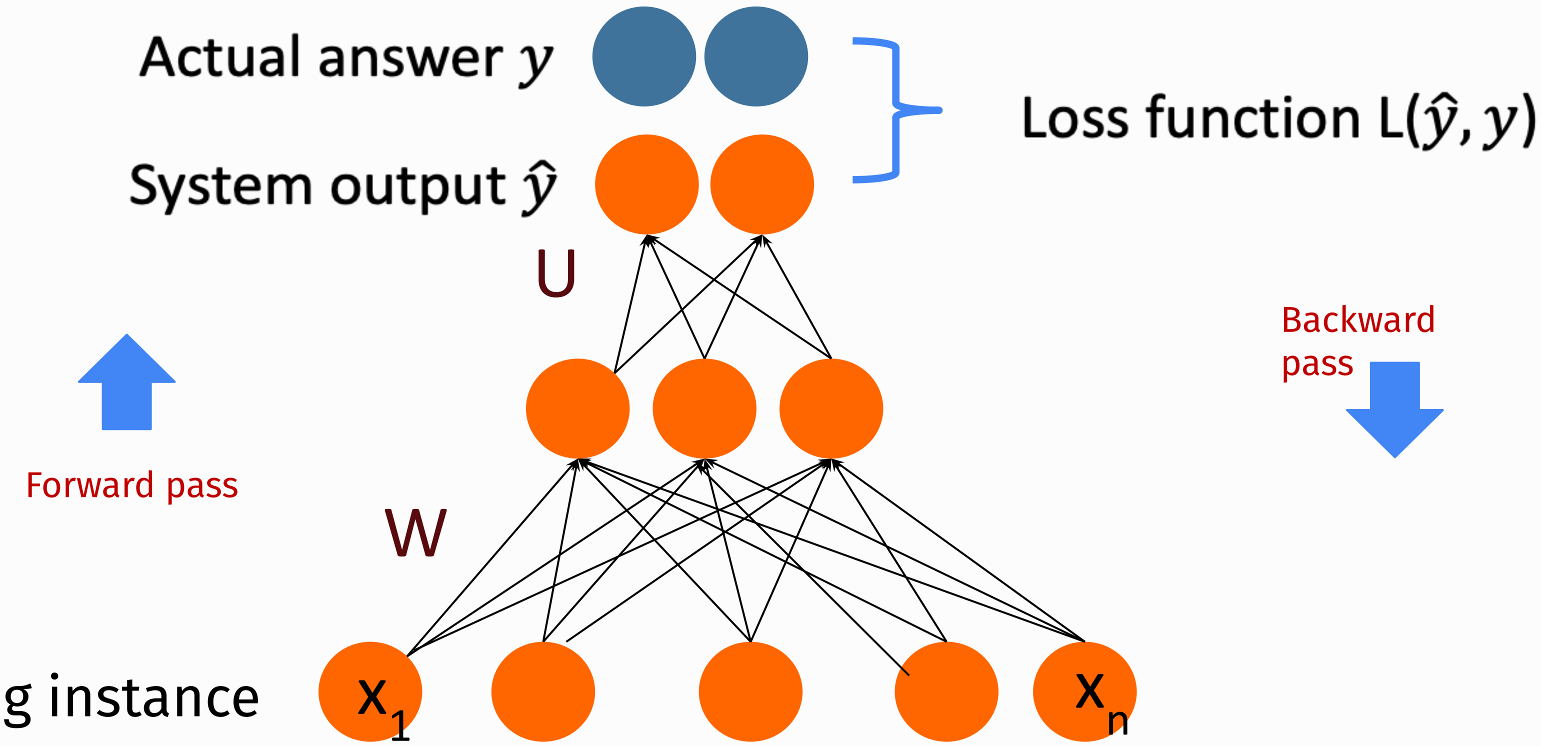
$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{k} e^{z_j}} \quad 1 \le i \le D$$



Slide adapted from Jurafsky & Martin

Training feedforward neural networks

Intuition: training a 2-layer Network



Slide adapted from Jurafsky & Martin

Remember stochastic gradient descent from the logistic regression lecture—find gradient and optimize

Slide credit: David Mortensen

The Intuition Behind Training a 2-Layer Network

For every training tuple (x, y)

1. Run **forward** computation to find the estimate $\hat{y}$
2. Run **backward** computation to update weights
   - For every output node
     - Compute the loss L between true y and estimated $\hat{y}$
     - For every weight w from the hidden layer to the output layer: update the weights
   - For every hidden node
     - Assess how much blame it deserves for the current answer
     - From every weight w from the input layer to the hidden layer
     - Update the weight

Slide credit: David Mortensen

Computing the gradient requires finding the derivative of the loss with respect to each weight in every layer of the network.

Error backpropagation through computation graphs.

Slide credit: David Mortensen

Reminder: gradient descent for weight updates

Use the derivative of the loss function with respect to weights $\frac{d}{dw} L(f(x; w), y)$

To tell us how to adjust weights for each training item

- Move them in the opposite direction of the gradient

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta \frac{d}{dw} L_{CE}(f(\mathbf{x}; \mathbf{w}), y)$$

- For logistic regression

$$\frac{\partial L_{\mathrm{CE}}(\hat{y}, y)}{\partial w_j} = [\sigma(w \cdot x + b) - y] x_j$$

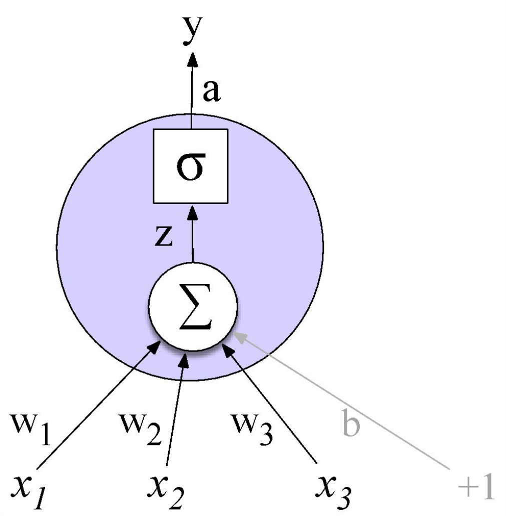
Slide adapted from Jurafsky & Martin

Where did that derivative come from?

Using the chain rule! $f(x) = u(v(x))$

Intuition (see the text for details)

$$\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$



Derivative of the weighted sum

Derivative of the Activation

Derivative of the Loss

$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial w_i}$$

Slide adapted from Jurafsky & Martin

How can I find that gradient for every weight in the network?

These derivatives on the prior slide only give the updates for one weight layer: the last one!

What about deeper networks?

- Lots of layers, different activation functions?

Solution:

- Even more use of the chain rule!!
- Computation graphs and error backpropagation!

Slide adapted from Jurafsky & Martin

Why Computation Graphs

For training, we need the derivative of the loss with respect to each weight in every layer of the network

- But the loss is computed only at the very end of the network!

Solution: **error backpropagation** (Rumelhart, Hinton, Williams, 1986)

- Relies on **computation graphs**.

Computation Graphs

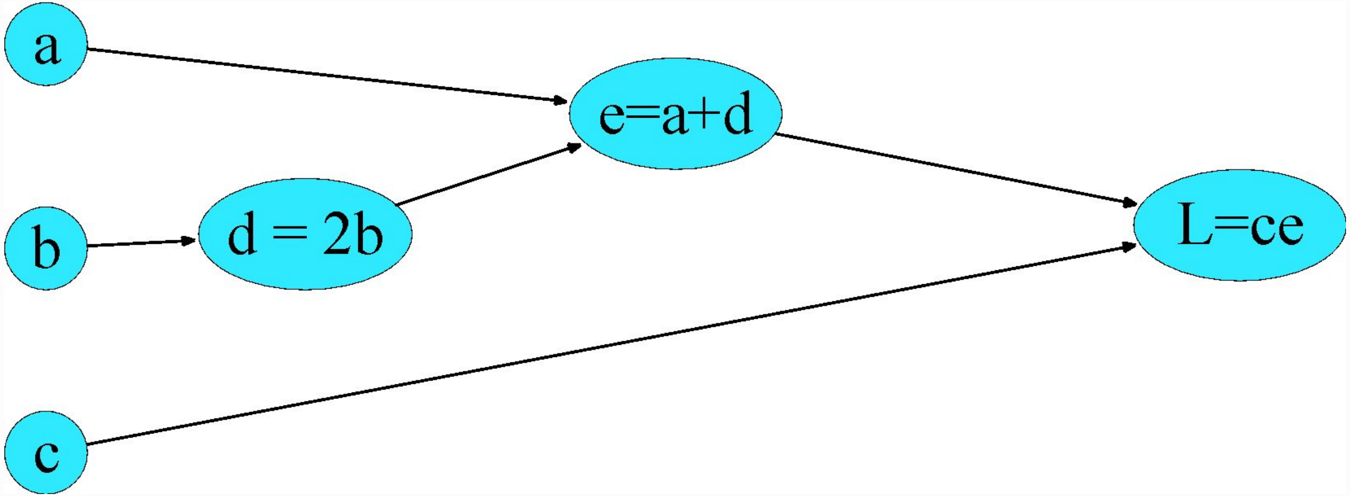
A computation graph represents the process of computing a mathematical expression

Slide adapted from Jurafsky & Martin

Example: $L(a,b,c) = c(a+2b)$

Computations:

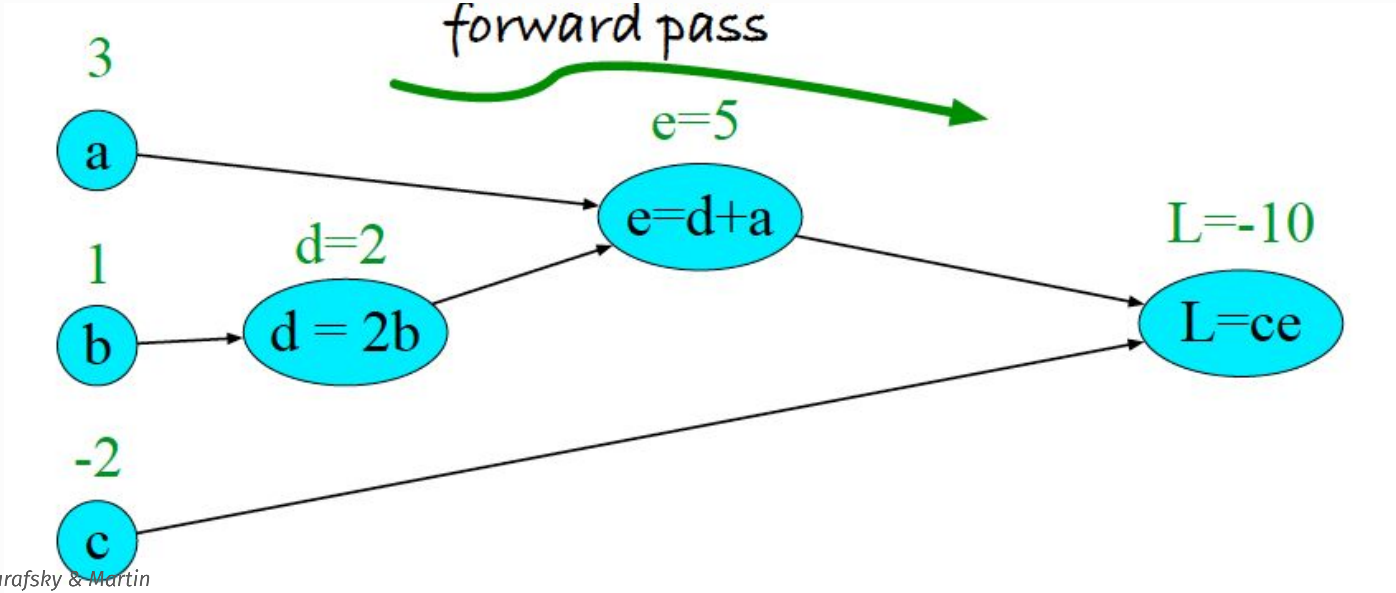
$$\begin{aligned} d &= 2*b \\ e &= a+d \\ L &= c*e \end{aligned}$$



Example: $L(a,b,c) = c(a+2b)$

Computations:

$$
\begin{aligned}
d &= 2 * b \\
e &= a + d \\
L &= c * e
\end{aligned}
$$



Backwards differentiation in computation graphs

- The importance of the computation graph comes from the backward pass
- This is used to compute the derivatives that we'll need for the weight update.
- How does a small change in that weight affect the final loss?

Slide adapted from Jurafsky & Martin

Example

$$L(a,b,c) = c(a+2b)$$

$$\begin{aligned} d &= 2*b \\ e &= a+d \\ L &= c*e \end{aligned}$$

We want: $\frac{\partial L}{\partial a}$, $\frac{\partial L}{\partial b}$, and $\frac{\partial L}{\partial c}$

The derivative $\frac{\partial L}{\partial a}$, tells us how much a small change in *a* affects *L*.

Slide adapted from Jurafsky & Martin

The chain rule

Computing the derivative of a composite function:

$f(x) = u(v(x))$

$$\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$

$f(x) = u(v(w(x)))$

$$\frac{df}{dx} = \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$$

Slide adapted from Jurafsky & Martin

Example

$$L(a,b,c) = c(a+2b)$$

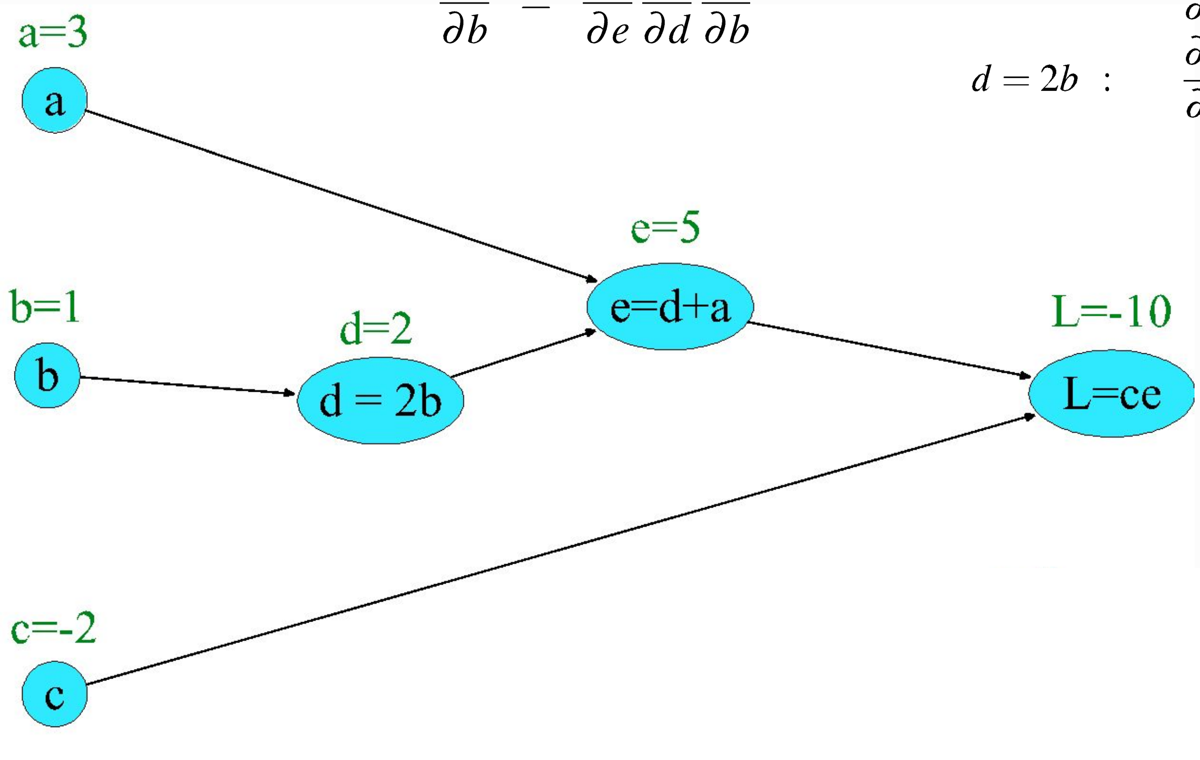
$$\begin{aligned} d &= 2*b \\ e &= a+d \\ L &= c*e \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial c} &= e \\ \frac{\partial L}{\partial a} &= \frac{\partial L}{\partial e}\frac{\partial e}{\partial a} \\ \frac{\partial L}{\partial b} &= \frac{\partial L}{\partial e}\frac{\partial e}{\partial d}\frac{\partial d}{\partial b} \end{aligned}$$

Example

$$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial e}\frac{\partial e}{\partial a}$$
$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial e}\frac{\partial e}{\partial d}\frac{\partial d}{\partial b}$$

$$L = ce \ : \quad \frac{\partial L}{\partial e} = c, \frac{\partial L}{\partial c} = e$$
$$e = a + d \ : \quad \frac{\partial e}{\partial a} = 1, \frac{\partial e}{\partial d} = 1$$
$$d = 2b \ : \quad \frac{\partial d}{\partial b} = 2$$



Example

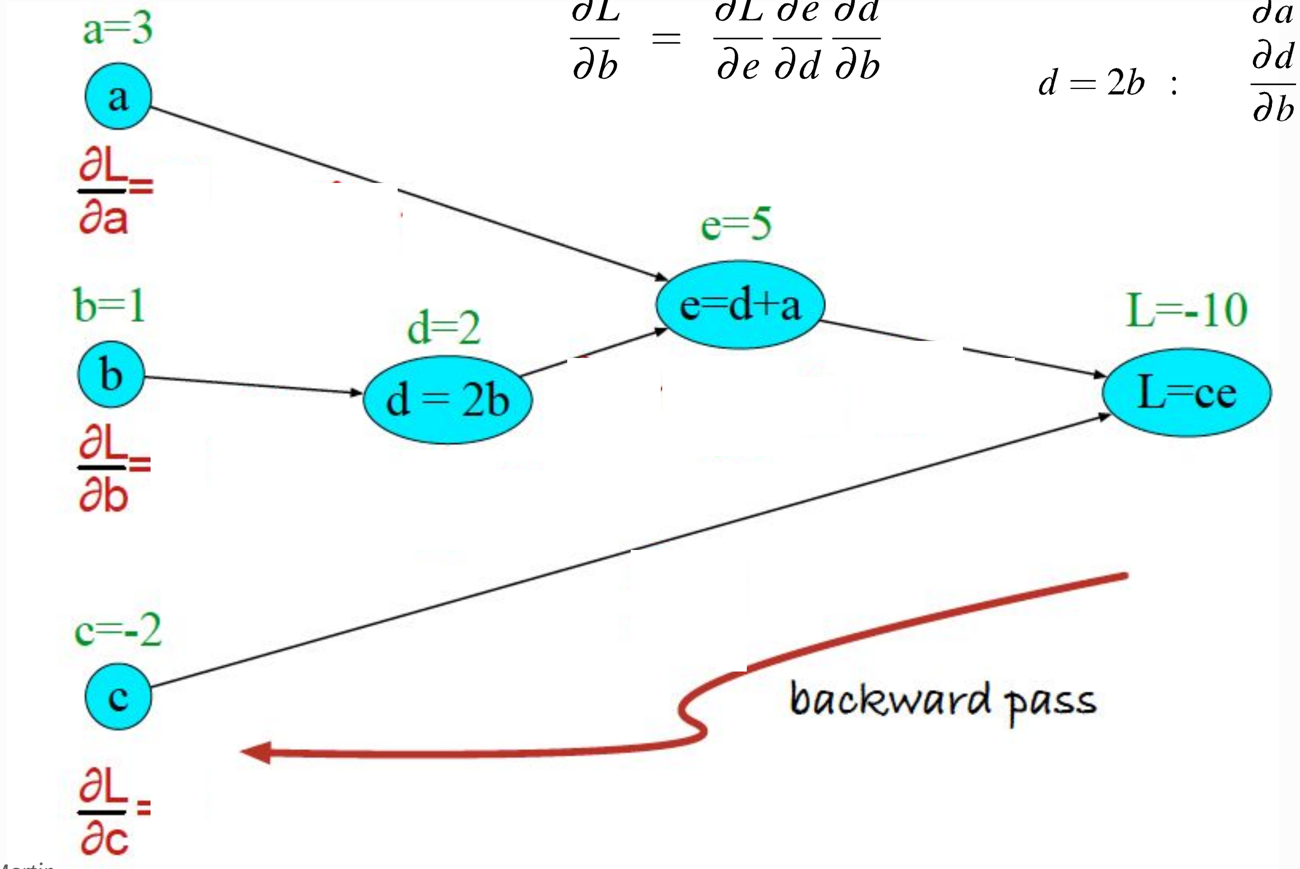
$$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial e}\frac{\partial e}{\partial a}$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial e}\frac{\partial e}{\partial d}\frac{\partial d}{\partial b}$$

$$L = ce \;:\; \frac{\partial L}{\partial e} = c, \frac{\partial L}{\partial c} = e$$

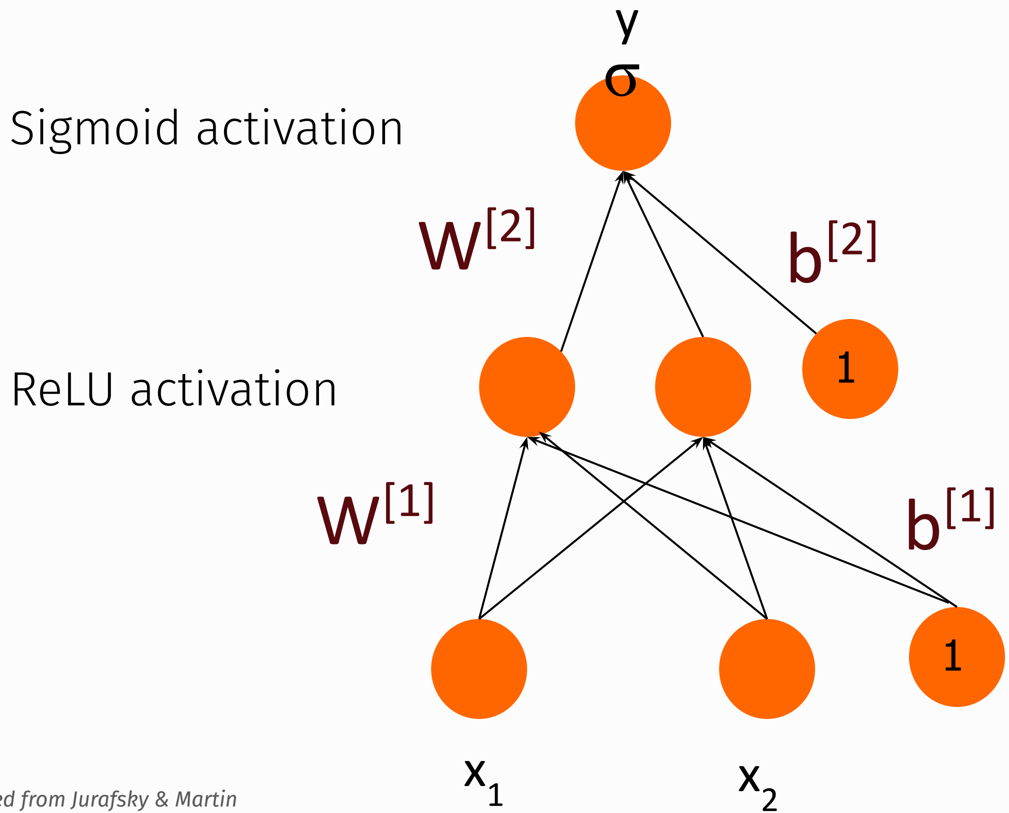
$$e = a + d \;:\; \frac{\partial e}{\partial a} = 1, \frac{\partial e}{\partial d} = 1$$

$$d = 2b \;:\; \frac{\partial d}{\partial b} = 2$$



Slide adapted from Jurafsky & Martin

Backward differentiation on a two layer network



$$
\begin{aligned}
z^{[1]} &= W^{[1]}\mathbf{x} + b^{[1]} \\
a^{[1]} &= \mathrm{ReLU}(z^{[1]}) \\
z^{[2]} &= W^{[2]}a^{[1]} + b^{[2]} \\
a^{[2]} &= \sigma(z^{[2]}) \\
\hat{y} &= a^{[2]}
\end{aligned}
$$

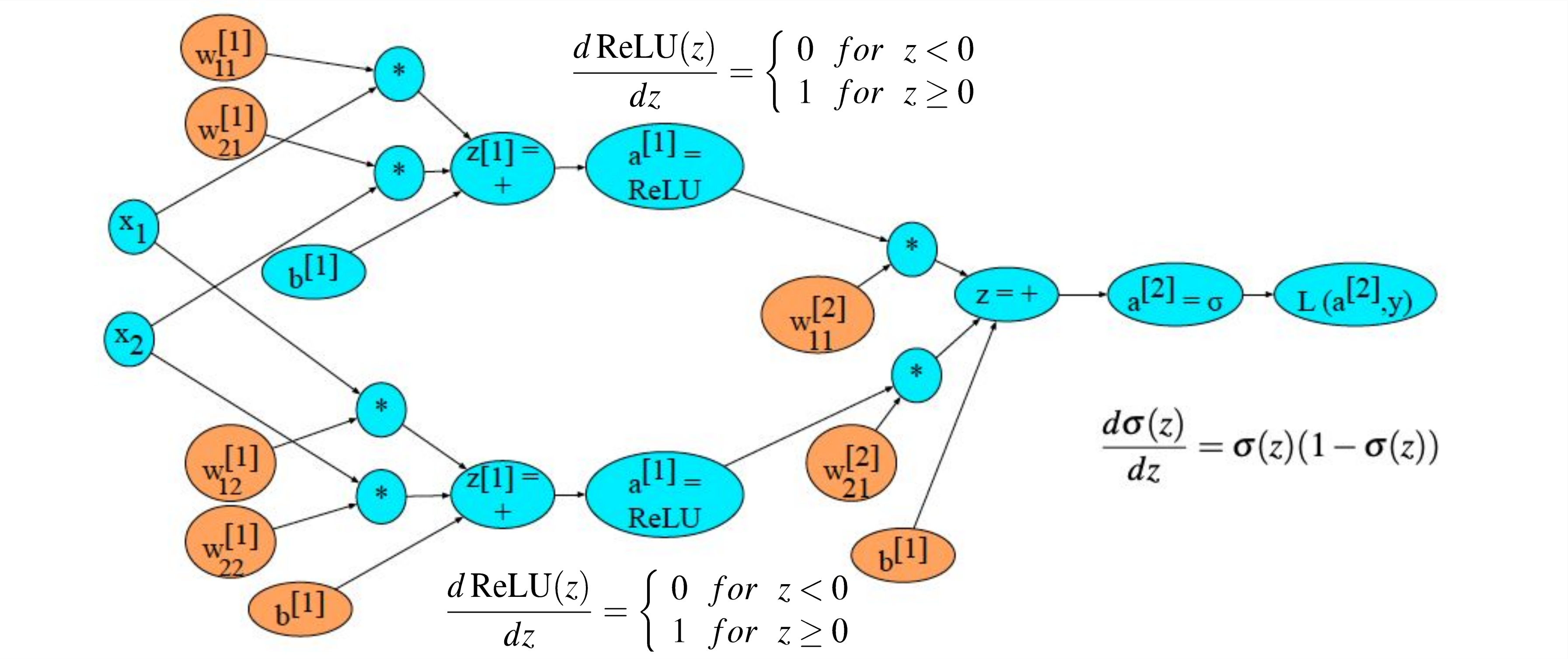
Backward differentiation on a two layer network

$$
\begin{aligned}
z^{[1]} &= W^{[1]}\mathbf{x} + b^{[1]} \\
a^{[1]} &= \mathrm{ReLU}(z^{[1]}) \\
z^{[2]} &= W^{[2]}a^{[1]} + b^{[2]} \\
a^{[2]} &= \sigma(z^{[2]}) \\
\hat{y} &= a^{[2]}
\end{aligned}
$$

$$\frac{d\,\mathrm{ReLU}(z)}{dz} = \begin{cases} 0 & for \;\; z < 0 \\ 1 & for \;\; z \geq 0 \end{cases}$$

$$\frac{d\sigma(z)}{dz} = \sigma(z)(1-\sigma(z))$$

Backward differentiation on a two layer network



Summary

For training, we need the derivative of the loss with respect to weights in early layers of the network

- But loss is computed only at the very end of the network!

Solution: **backpropagation**

Given a computation graph and the derivatives of all the functions in it we can automatically compute the derivative of the loss with respect to these early weights.

Questions?